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Georeferencing and Distortion Analysis of the 1:75 000 Czech Special Maps of the Third Military Survey

1. The III rd Military Survey of the Czech Countries

The third historical Military Survey concerning the territory of the Czech Crown, was carried out in the Austria-Hungarian Imperium in the period of 1870-1883, there of in Bohemia 1877-1879, in Moravia 1876-1877 and in Silesia 1876. The survey was carried out at decimal scale 1:25 000 and resulted in topographic sections. Numeric base for the Survey were trigonometric stations in coordinate systems of Gusterberg and St. Stephen transformed into the polyedric projection. The graphic base consisted of cadastral maps reduced into scale 1:25 000. Four topographic sections (TS25) represented one sheet of Special Map 1:75 000 (SM75) with dimensions of 30´ geographic longitude and 15´ geographic latitude. The sheets of SM75 were labelled by number of layer (Arabic figures), number of column (Roman figures) and name of significant settlement, e.g. 5-X Kladno. The layers were numbered starting from parallel 51° 15´ southwards and the columns starting from meridian 27° from Ferro eastwards. After 1917 a new labelling was introduced that is valid until now.

The Maps of the III rd Military Survey survived thanks to their importance the end of the Austro-Hungarian Imperium in 1918 and became the state map series in Austria, Hungary and Czechoslovakia.

In 1919 the Military Geographic Institute in Prague was founded that took over from the Austrian Military Geographic Institute the cartographic records of the III rd Military Survey for the territory of Bohemia, Moravia, Silesia, Slovakia and Subcarpathian Ruthenia. Altogether 699 Topographic Sections, 189 sheets of the Special Map and 33 sheets of the General Map all of them in the form of printing documents, i.e. copper printing plates.

The German and Hungarian geographical names on the maps were replaced by the Czech and Slovak one. The contents of the SM75 was gradually upgraded by green filling of forest areas and by input of kilometre grid of the coordinate system S-JTSK in the Krovak projection used in Czechoslovakia since 1927 until today.

Especially the SM75 enjoyed a great popularity and was produced in several thematic series. And even today the SM75 map is very popular in Czechia. In 1956 the maps of the III rd Military Survey were substituted by a unified military map series of the Warsaw Pact Countries. These maps have been produced in S-42 coordinate system and in Gauss–Krüger projection. Their validity expired by the 31st December 2005 when the Czech Topographic Series switched over to NATO standards, i.e., ellipsoid WGS84 and UTM/WGS84 projection. Importance of the III rd Military Survey maps for studies of countryside development and introduction of new technologies in cartography is enormous an a great attention should be therefore dedicated to their study.

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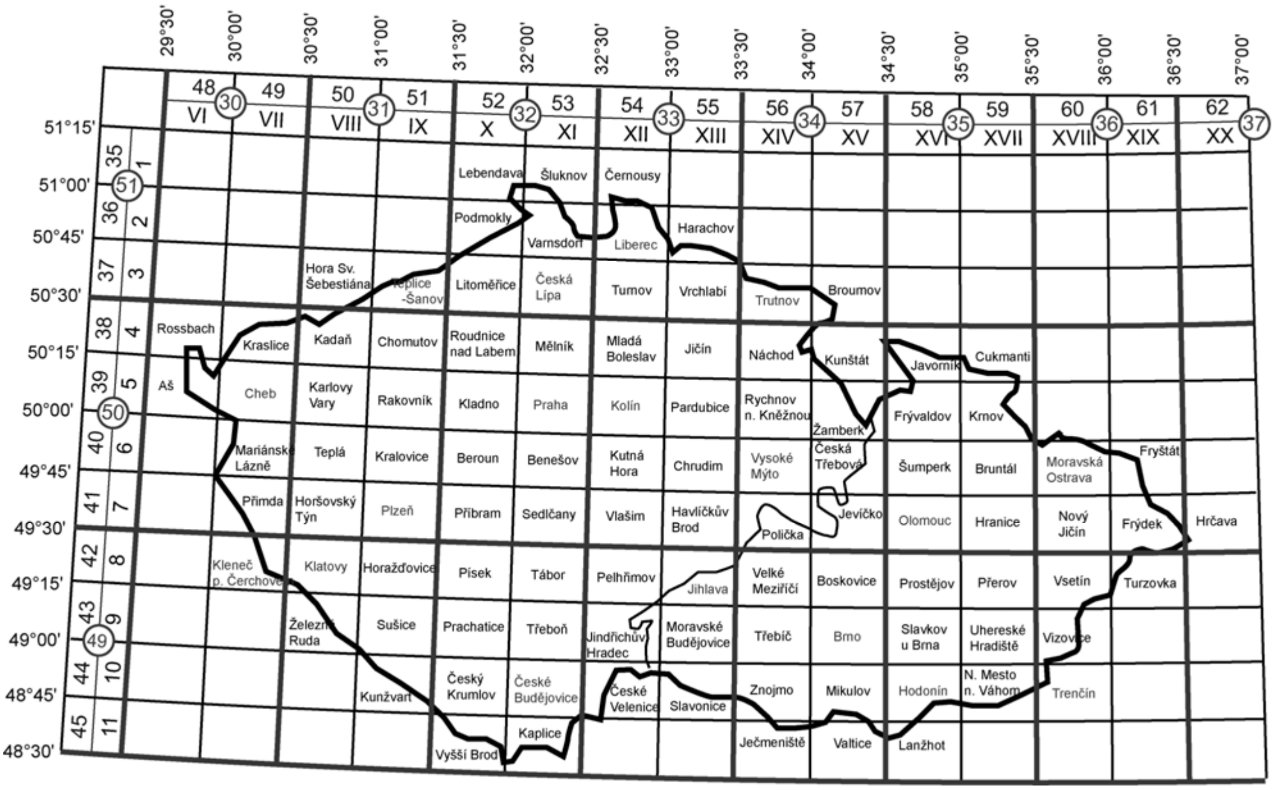

Fig 1. Index of SM75 maps - Czech Republic

2. Mathematical theory of Krovak projection

S-JTSK is plane geodetic grid coordinate system called in Czech as "System **J**ednotne **T**rigonometricke **S**ite **K**atastralni - The System of the Unified Czech/Slovak Trigonometric Cadastral Net". Scale, location and orientation of the S-JTSK on the surface of the Bessel ellipsoid was derived from the results of the historical Austro/Hungarian military surveys in the years 1862-98. There are 42 identical points on the Czech territory used for transformation computations. Astronomical orientation was measured only on the Hermannskogel, a trigonometric point in Austria. Krovak projection and national grid S-JTSK were adopted on the territory of the Czech and Slovak republics (former Czechoslovakia) in 1927. The using of this system for all geodetic surveying and cartographic activities (state mapping) is in context with the Czech State Law Nr. 200/94.

The Conformal Oblique Conic Projection of Czechoslovakia was prepared by Josef Krovak in the year 1922 for construction of cadastral maps and topographical maps of medium scales for the civil geodetic service of Czechoslovakia. The Bessel ellipsoid of 1841 ($a = 6\,377\,397.155$ m, $1/f = 299.152812853$) which is widely adopted in Central Europe, is used. Longitude λ refers to the Ferro meridian (an island of the Canaries), not the Greenwich meridian ! The projection is conformal, so that in the projection plane meridians and parallels intersect at right angles. Round value 17^{o} 40' is used for cadastral and topographic mapping in the Czech Republic/Slovakia and Austria. The rectangular plane grid S-JTSK has an origin in Finnish Basin, near Tallinn. The X-axis normally coincides with the meridian 42^{o} 30' from Ferro increasings south. The Y-axis is perpendicular to the X-axis and increasings to west.

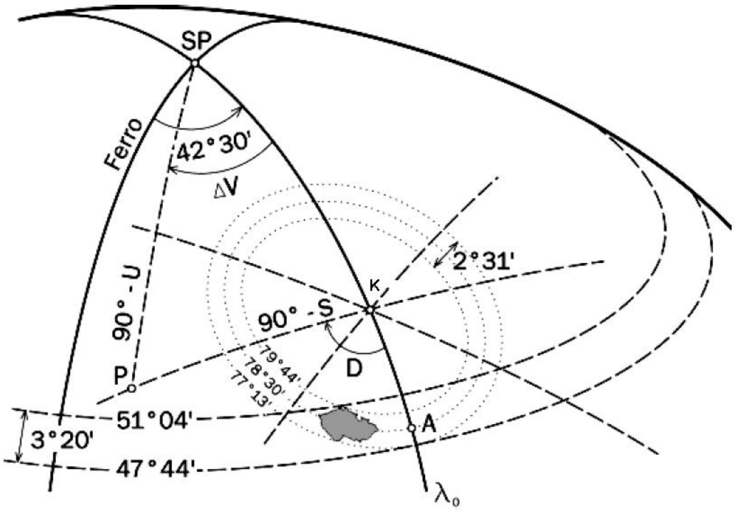

Fig 2. Krovak projection – localization of the Czech Republic on the Bessel Ellipsoid (Talhofer 2007)

Note that this projection consists of four steps.

a) Conformal projection of Bessel ellipsoid to Gauss sphere

Projection ellipsoid - sphere is given by the relation T_1

$$T_1: \ (\varphi,\lambda_{ferr\,o}) \rightarrow (U,V)$$

Where the input values are

φ : latitude on the Bessel ellipsoid

λ_{ferro} : longitude on the Bessel ellipsoid from Ferro

and the output values are

U : latitude on the Gauss sphere, V : longitude on the Gauss sphere

The general formula may be written as follows:

$$U = f(\varphi),$$

$$V = g(\lambda_{ferro})$$

but since we use the conic projection, relation (2.2) may be inverted into

$$\mathrm{tg}\,(0.5\,U + 45^{o}) = k\,\{\mathrm{tg}^{\alpha}(0.5\,\varphi + 45^{o}) * ((1-e\sin\varphi)/(1+e\sin\varphi))^{\alpha e/2}\}$$

and for the value V

$$V = \alpha * \lambda_{ferro}$$

with the following constants

k = 1.003419164, α = 1.000597498372, e = 0.0816968303

b) Change from geographic to cartographic coordinates on Gauss Sphere

Purpose : the change from the North Pole (S_p) to the Cartographic Pole (Q) with a respect to the borderline of the former Czechoslovak Republic. There is no change of Gauss sphere surface. This transformation is given by the relation T_2

$$T_2 : (U,V) \rightarrow (S,D)$$

where spherical triangle should be solved with values

S - cartographic latitude on the Gauss sphere (from pole Q)

D - cartographic longitude on the Gauss sphere (from pole Q)

where the so called cartographic pole $Q\ (\varphi_Q , \lambda_Q)$ on the Bessel ellipsoid has coordinates

$\varphi_{\mathbf{Q}} = 59^o\ 45'\ 27''$, $\lambda_Q = 42^o\ 30'\ 00''$

and on the Gauss sphere as a pole $Q\ (U_Q ,V_Q)$, with the numerical values

$U_Q = 59^o\ 42'\ 42.69689''$, $V_Q = 42^o\ 31'\ 31.41725''$

where the transformation relation T_2 is determined by formulas

$$\sin(S)=\cos(90^o-U_Q)*\sin(U)+\sin(90^o-U_Q)*\cos(U)*\cos(V_Q-V)$$

$$\sin(D) = [\ \cos(U) / \cos(S)\ ] * \sin(V_Q-V)$$

c) Conformal projection of Gauss Sphere to the oblique tangent cone

Purpose : wrapping an oblique cone around the sphere, so that it touches the Gauss sphere surface along the great circle (standard cartographic parallel $S_o = 78^o\ 30'$) where no distance distortion is assumed. This projection is given as relation T_3

$$T_3 : (S,D) \rightarrow (\rho, \varepsilon) ,$$

where

$$\rho = \rho_o\ \{\mathrm{tg}\ (0{,}5\ S_o + 45^o) / \mathrm{tg}\ (0.5\ S + 45^o)\}^n$$

$$\varepsilon = n * D ,$$

with the constants $\rho_o = 1\ 298\ 039.0046$, $S_o = 78^o\ 30'$, $n = \sin(S_o) = 0.9799247046$

where the polar coordinate $\boldsymbol{\varepsilon}$ is an angular distance and second polar coordinate ρ is a radiusvector from the cartographic pole.

d) Projection of oblique cone to grid plane S-JTSK

Polar conic coordinates (ρ,ε) are transformed onto rectangular coordinates (X,Y) using relation T_4. As was mentioned in the previous text, the origin of the grid X,Y system called S-JTSK is a projection of the cartographic pole Q on the plane. This point lies near Estonian Tallinn in the Finnish Basin. The X axis coinciding with the meridian of longitude $\lambda_{ferro}=42^o30'$ increasing to south, the Y axis perpendicular increasing to west. The whole Czechoslovak territory is situated only in the first quadrant with positive coordinates.

Final relation T_4 between polar and rectangular (grid S-JTSK) coordinates is given by

$$T_4 : (\rho, \varepsilon) \rightarrow (X,Y) ,$$

where

$$X = \rho * \cos \varepsilon$$

$$Y = \rho * \sin \varepsilon ,$$

where the coordinates $(Y,X)_{\text{S-JTSK}}$ are the resulting coordinates searched for.

3. Georeferencing of SM75 sheets in Krovak projection

For computation of corners of special maps in Krovak projection was developed software MATKART – program. Authors of this software are Bohuslav Veverka from the Czech Technical University in Prague and Monika Cechurova from the University of West Bohemia in Pilsen. Computing of map sheet corner coordinates is very simply. User must only inreactively select scale – where 25 000 is topographic section, 75 000 special map or 200 000 general map and insert map index.
User screen with results of computations is on the Fig. 3.

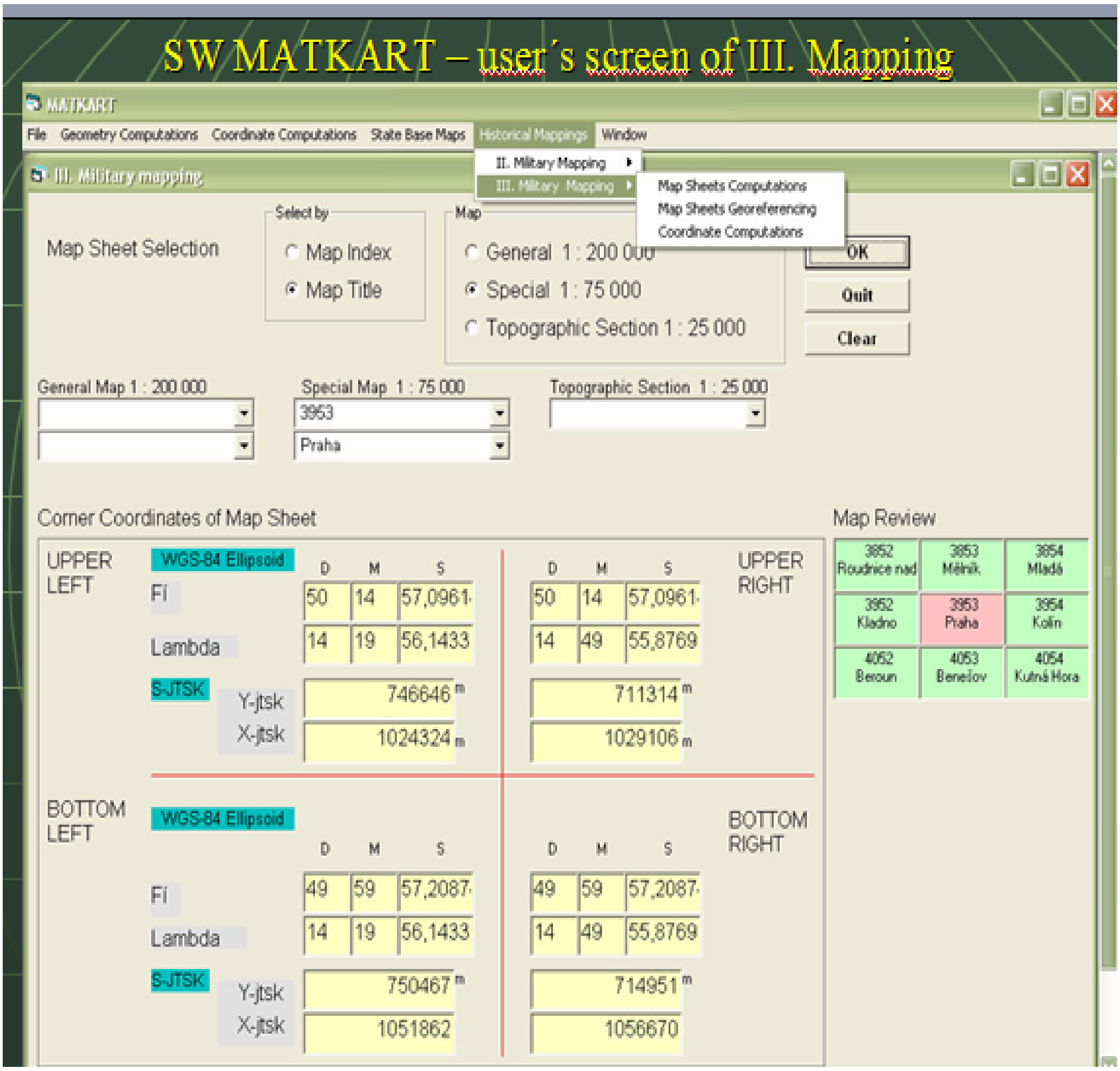

Fig. 3 MATKART – computation of corners of Special Map 3953 Praha

4. Influence of distortion of the SM75 map and its elimination by means of Helmert Transformation

We consider a scanned image of map SM75 a we know the raster coordinates of corners. We also presume to know the ideal coordinates of individual map corners. Further we also presume to know the ideal coordinates of individual map corners $(Y,X)_{S\text{-}JTSK.}$ As it is possible to derive from the labels of individual map sheets also the geographic coordinates of their corners $(\varphi,\lambda_{ferro})$ and calculate by the above mentioned procedure their ideal values, it is possible to use a suitable transformation for recalculation of coordinates obtained by raster digitisation.

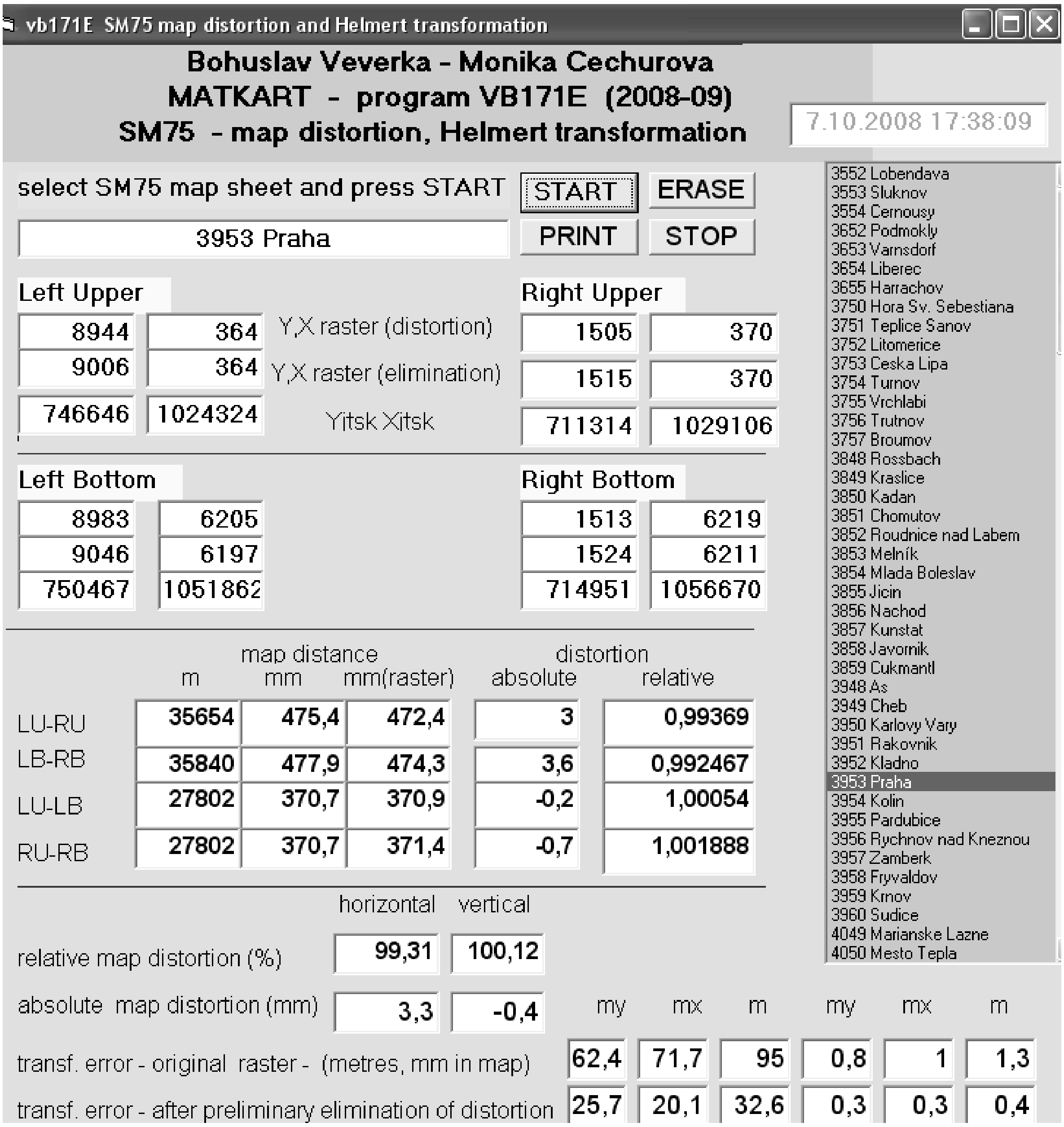

Fig 5. User´s screen of VB171E program – computation of the SM75 ideal size, solution of the map distortion (shrinkage) and transformation errors

5. Statistical Analysis of map distortion for the State Map Series SM75 of the Czech Republic

Using the Program VB171E a total number of 173 maps SM75 have been evaluated (see Donovalova 2009). The goal of this activity has been to find out extreme values of distortion, it means shrinkage of map sheets and namely suitability of Helmert Transformation for recalculation of raster coordinates of a map influenced by distortion into the S-JTSK system. The analysis has been carried out in an EXCEL environment with interesting results.

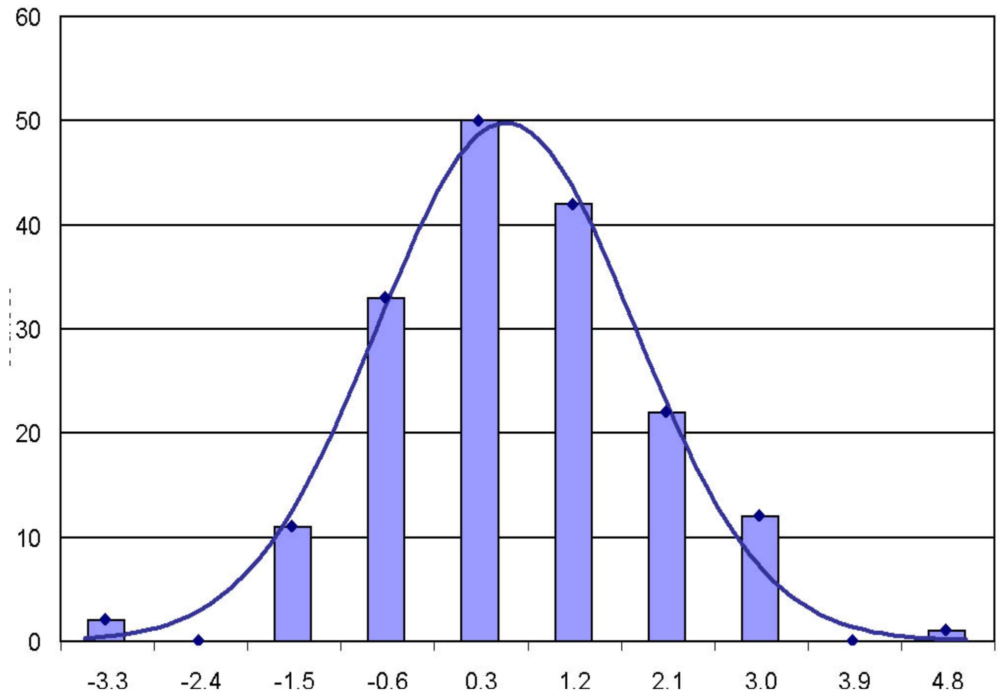

Fig 6. Normal distribution of absolute horizontal distortion – 173 map

From Fig 6. we can gain a relatively interesting knowledge that complex of 173 old historical paper SM75 maps covering whole territory of former Bohemia, Silesia, Moravia and Slovakia has practically perfect normal Gaussian distribution.

By using of regress analysis we are able to study relation between results of Helmert transformation without preliminary elimination of shrinkage – see Fig. 7 where is a strong linear relation m_y and m_x and Fig. 8 after elimination of influence of distortion.

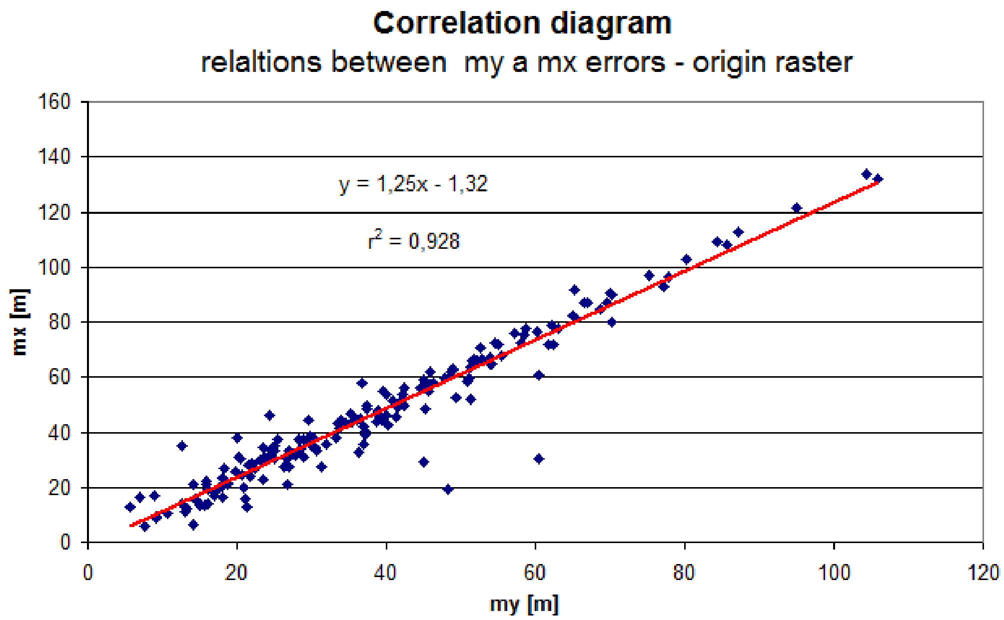

Fig. 7. Correlation diagram of relations between errors m_y and m_x Helmert transformation without preliminary reducing of map distortion

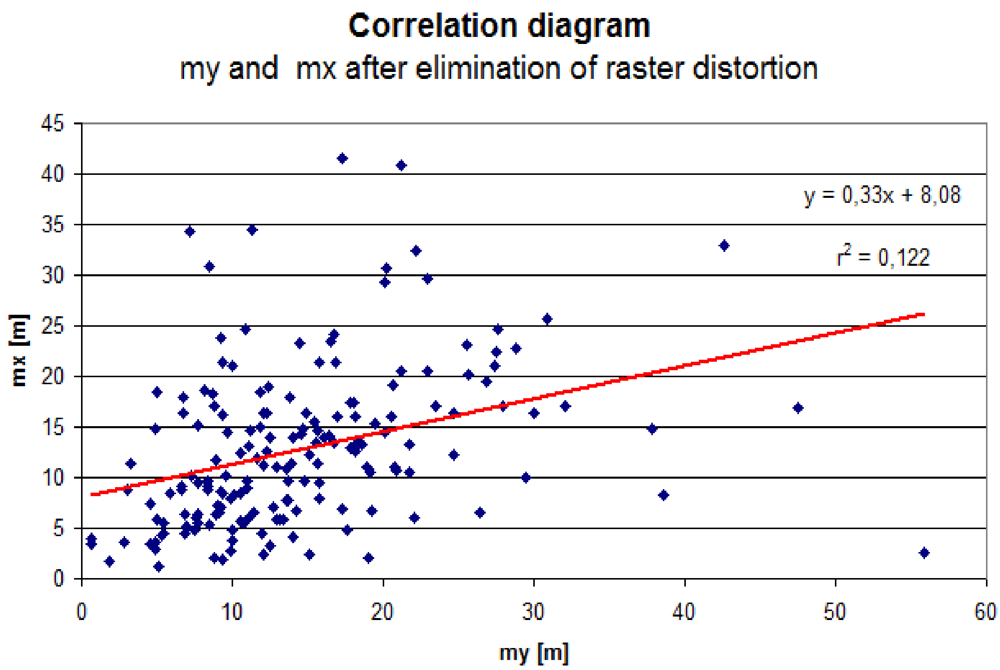

Fig. 8. Correlation diagram of relations between errors m_y and m_x Helmert transformation after preliminary reducing of map distortion

6. Conclusion

This results proves that even the relatively old maps as SM75 sheets of about 80 years and printed on paper of low quality can be successfully converted into state information coordinate system like S-JTSK for Czech and Slovak Republic.
Errors of manual interactive digitization of map sheet corners and errors due to non-homogenous shrinkage may be eliminated by procedure described in this contribution.

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