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BASICS OF FUZZY LOGIC AND ITS APPLICATION IN GEOLOGY

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Master's thesis

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Declaration

I hereby declare that this master's thesis is my own work and all information resources are scheduled in the list of references in accordance with guidelines on the ethical preparation of university thesis.

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In Prague, 12.5.2010 Libuše Vejrová

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Abstract

The master’s thesis introduces into the problematic of fuzzy set theory, both the comparison the fuzzy logic with the classical (Boolean) logic, and analysis of its principles. The logic propositions are enriched by a membership degree, because the fuzzy elements may belong to the set not only wholly, how it is used in classical logic, but as well as partially. Thereby the basic operations with fuzzy sets differ with crisp sets. With fuzzy logic we are encountering in normal life. Always with expressed hesitating sentence to any problem, it is kind of fuzzy logic, whether it is a matter of hunger, a matter of size of people, accuracy of measures, or thinking of the washing machine about dirty of cloths. Geology is typical science, which can be hardly represented with classical classification methods, therefore this thesis deals with the problems of visualization of geological fuzziness on map.

Key words

Uncertainty, fuzzy set theory, fuzzy logic, membership function, fuzzy inference system, Mamdani – Assilian method, Takagi – Sugeno method, geological mapping, rock.

Abstrakt

Diplomová práce uvádí do problematiky teorie fuzzy množin, jak porovnáním fuzzy logiky s klasickou (Booleovskou) logikou, tak i rozborem svých principů. Logické výroky jsou obohaceny o stupeň příslušnosti, neboť fuzzy prvky mohou do množiny patřit nejen zcela, jak je to běžné u clasické logiky, ale i částečně. Tímto se základní operace s fuzzy množinami liší od operací s klasickými množinami. S fuzzy logikou se setkáváme v běžném životě. Při váhavém postoji k jakékoli problematice se jedná o fuzzy logiku, ať už je to otázka hladu, velikosti člověka, přesnost měření, nebo přemýšlení pračky o špinavosti prádla. Geologie je typický obor, který může být jen stěží reprezentován tradičními klasifikačními metodami, proto se tato práce zabývá problematikou vizualizace geologických nepřesností v mapě.

Klíčová slova

Nepřesnost, teorie fuzzy množin, fuzzy logika, funkce příslušnosti, systém fuzzy řízení, Mamdani – Assilianova metoda, Takagi – Sugenova metoda, geologické mapování, hornina.

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Chapter 1

Introduction

As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.

Albert Einstein

In many human activities fuzzy set theory is used, or rather begins to be used. It started in the general theory of systems and in control engineering. Nowadays the fuzzy sets are used in economics, medical diagnostics, in estimation of the stability of building structures for earthquake prediction, for description of the action of chemical reactors, but also in library science and finance.

The practical application is more attractive and companies from the Far East supply the World with fuzzy processors used in “intelligent” washing machines, vacuum cleaners, cameras and other various equipment. The question is, whether this is a matter of fashion trend, which will have only a transitory duration, or if this concern signalizes some certain changes in the way of our thinking. Today many of the arguments suggest that this is a fundamental change, which we could designate as moving or change of the pattern, to whereby adhere the deeply rooted stereotypes of our thinking.

Fuzzy approach is not only a kind of new approach to the expression of uncertainty, which has interesting technical applications, but it inverts the way of thinking, which has been used in western civilization since ancient times.

From the Aristotle period, we learn in logic, that the claim can be either true or false, there is no third option. In so-called exact science disciplines, we try to verify the truthfulness of our claims by objective measurements. Galileo gave to modern physics his motto:

Measure what can be measured, and make measurable what cannot be measured.

The precise quantitative description of natural phenomena based on this approach enabled the utilization of mathematical methods for the systematization of observed data and to build theories, which are, in terms of explanation and also prediction of these phenomena, very effective.

Chapter 2 shortly explains the development of the fuzzy set theory and introduces the problems of fuzzy logic, which does not include probabilistic reasoning.

The principles of fuzzy logic are analyzed in Chapter 3, where the conception of fuzzy set is explained, and how the fuzzy logic differentiates from classical Boolean logic. Because fuzzy set is not only true or false, the very important part of fuzzy set theory is membership and definition of the membership function. In Chapter 3, fuzzy operations with fuzzy sets will also be discussed.

Principles of fuzzy logic are closely related to expert systems, which are explained in Chapter 4, the same chapter will also describe the Mamdani – Assilian inference systems, Takagi – Sugeno inference systems and differences between them.

This thesis deals with very fuzzy logic, because it is new and progressive discipline, which tries to be utilized in lot of trends.

Because one of the aims of this thesis is to apply the fuzzy set theory into the specific brunch – geology, the inseparable part is short description of geological mapping. Because geology is a very extensive science, it is not possible to give accounts of all principles during mapping and sequentially draw the mapped phenomena into the map. Chapter 5 only outlines the procedure and problems of geological mapping. And in Chapter 6 is described the model of geological maps and problems with depicting them.

Chapter 2

Introduction to fuzzy set theory and fuzzy logic

2.1 Introduction

Fuzzy set theory is a relatively young part of mathematics. Those who participated in its development include Lofti A. Zadeh, Jan Lukasiewicz as well as others. In order to explain fuzzy set theory, we should start with the classical set. Classical set theory provides mathematics formalism which is successfully used for describing many concrete situations and for huge generalization which allows a unique approach to solving problems. The basic principle of classical set theory is the fact that each admissible element either belongs to or does not belongs to the set. Other options cannot occur and both options cannot occur at the same time. If the element appertains to the set, it is possible to express by quantity, which uses the values of 1 or 0. This two-valued logic (Boolean logic) is obvious and easy for computing, but in many cases is not appropriate for describing the real world. We can make certain of that in many paradoxes, which are often known from ancient times.

One of them is the sorites paradox, or “the paradox of the heap”. It is commonly accepted that a large quantity of grains of sand is called a heap. According to the classical set theory we initiate an element *Group of grains of sand* and a set of *Heap*, which includes all the groups of grains of sand, which are known as heap. But what happens when one grain of sand is taken away? If this were to occur, we would still have a heap. What happens if we keep repeating this operation over and over again? Then we get down to a situation, where there is left only one single grain of sand. Naturally, this is no longer called a heap. During the removing of grains of sand we have entered into a situation where the previous group of grains has been the heap and new group of grains is no longer the heap. It speeds badly to establish, in which step the heap of sand ceases to be the heap after removing only one grain. The same problem occurs in the opposite procedure. From the amount of grains of sand the heap does not set in with addition of one grain. Both these situations are caused by the effort of describing the real world by the help of two values. The group of grains of sand is or is not the heap. Any other option is not permitted. This implies that the characteristic "being the heap" cannot be classical, two-valued. It turns out, that people used to comprise into their account the characteristics, which are vague, more or less valid, without sharp borders. The very type of vagueness is strived to describe by fuzzy logic.

A more practical example of the need to replace sharp boundaries with fuzzy boundaries is browsing a big database.

In the 19th century the French police officer Alfonse Bertillon proposed a system for identifying criminals according to physical characteristics [Navara & Olešák, 2007]. His method had been used in criminal science. If nowadays this method was used, data would surely be gathered in a database, where we search according to different criterion (combined to complicated logical expressions). For each parameter, we would need to choose a definite tolerance, which would cover not only uncertainty of measure, but inconstancy of constancy as well, e.g. flesh or stature. If we set the tolerance too small, the selection may not contain the guilty person. If the tolerance is too big, the selection is too large and useless. It is very difficult to set the tolerance for each variable. It would be better to think about valuation, that "small" deviation implicates small reduction of value, then "big" deviation has more substantial impact, but still does not have to eliminate appropriate alternative. It is still possible to formulate strict rules, how each deviation is assessed. Finally, it remains to establish the combination of deviation to more complicated logical expression. With this approach one can take advantage of a maximum of information and have too large selection, which is ordered after assessment.

Fuzzy logic appeared for the first time in 1965 in an article, which was published by Professor Lotfi A. Zadeh. In that time, basic notion of fuzzy logic were defined, the basic notion is the so-called fuzzy set. The term *fuzzy* represents an effort of fuzzy theory to coverage of reality in its uncertainty and inaccuracy.

A classical set theory is susceptible of only two states of element in the face of set. The element belongs into the set wholly or not at all. In the Figure 2.1, one can see that in the classical set theory the element x is not possible to file into the set A, neither the set B. Fuzzy logic extends this concept. The grade of membership is expressed in the scale of 0 to 1 and it is a continuous function. Fuzzy set is a set, which supposes not only full or empty membership, but partial membership as well. A measure of membership of definite element in the fuzzy set is expressed with membership degree. The value of the membership degree

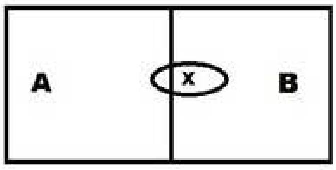


Figure 2.1: Submition the element x into the set A and the set B.

of each element is obtained by so-called membership function. Fuzzy logic is reasoning with fuzzy sets. Operations on fuzzy sets are similar to those of standard logic but are differently defined.

Probability theoreticians have presented the heaviest critique and that is the reason why many fuzzy logic authors (Zadeh [Zadeh, 1965], Klir [Klir & Folger, 1988], Kosko [Kosko, 1990] and other) have included the comparison between probability and fuzzy logic in their publications. Fuzzy researchers try to separate fuzzy logic from probability theory, whereas some probability theoreticians consider the fuzzy logic a probability in disguise.

Classical probability theory is not sufficient to express uncertainty encountered in expert systems. The main limitation is that it is based on two-valued logic. An event either occurs or does not occur, there is nothing in between. Another limitation is that in reality events are not known with sufficient precision to be presented as real numbers.

For probability we make out, if the phenomenon may occur and its probability, whereas for fuzzy logic we know, that the phenomenon exists and we occur its membership degree in definite set (can be called the verity of statement). For example we have red colour, but it might be a different shade. It is possible to express this tone with the membership degree in range of 0 – 1 (continuously). A function referring this value is called the membership function. Fuzzy approach is based on the premise that key elements in human thinking are not just numbers, but can be approximated to tables of fuzzy sets or classes of objects in which the transition from membership to non-membership is gradual rather than abrupt. In general, fuzzy concepts allow possibility and overlapping classes as a way of thought.

The sense of fuzzy logic can be perceived in two levels. One level is necessary to work with inaccurate or misty data. The second level is a general approach for describing the real world. When using accuracy description it happens to the idealization of reality and then diversion from reality.

The term *fuzzy logic* has two meanings. According to the first interpretation (in the narrower sense) it is seen as a multi-valued imprecise logic and as an extension to the more traditional multi-valued logic. Bart Kosko explains this point of view by emphasizing that in reality everything seems to occur or to be true to a degree. Facts are always vague, fuzzy or inaccurate to some extent.

"Only mathematics has black and white facts and it is only a collection of artificial rues and symbols. Science deals with gray or fuzzy facts as if they were black-and-white facts of mathematics. Nobody has presented a fact having to do with the real world that is 100 per cent true or 100 per cent false. They are said to be."

The first meaning was some kind of model for human reasoning. The other interpretation (in a larger sense) is that fuzzy logic is equal to fuzzy set theory. According to this view any field *X* can be fuzzified by changing a set in *X* by a fuzzy set [Zadeh, 1994]. For example, set theory, arithmetic, topology, graph theory, probability theory and logic can be fuzzified. This has already been done in neurocomputing, pattern recognition and mathematical programming.

The use of heuristic linguistic rules may be the most suitable resolution to this problem, if the conventional techniques of system analysis cannot be incorporated with success into the modelling or control problem. For example, there is no mathematical model for truck and trailer reversing problem, in which the truck must be guided from an arbitrary initial position to a desired final position. Humans and fuzzy systems can perform this nonlinear control task with relative ease by using practical and at the same time imprecise rules as "If the trailer turns smoothly right, then turn the wheel smoothly right".

2.2 Neural networks

The study of neural networks started with publication of Mc Culloch and Pitts [Mc Culloch & Pitts, 1943]. The single-layer networks, with threshold activation functions, were introduced by Rosenblatt [Rosenblatt, 1962]. These types of networks were called perceptrons. In the 1960s it was experimentally shown that perceptrons could solve many problems, but many problems which did not seem to be more difficult could not be solved. These limitations of one-layer perceptron were mathematically shown by Minsky and Papert in their book Perceptrons. The result of this publication was that the neural networks lost their interestingness for almost two decades. In the middle of the 1980s, back-propagation algorithm was reported by Rumelhart, Hilton and Williams [Rumelhart et al., 1986], which brought back the study of neural networks. The significance of this new algorithm was that multilayer networks could be introduced by using it.

The neural network makes an attempt to simulate the human brain. The simulation is based on the present knowledge of brain function and this knowledge is even at its best primitive. It is not completely wrong to promote that artificial neural networks probably have no close relationship to operation of human brains. The operation of the brain is believed to be based on simple basic elements called neurons which are related to each other with transferring lines called axon

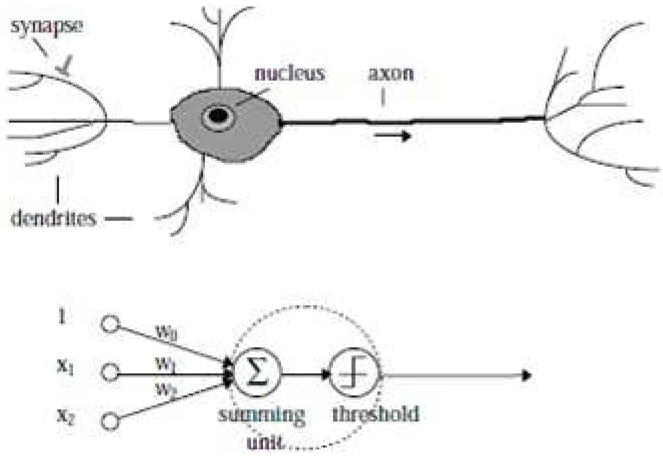


Figure 2.2: Simple illustration of biological and artificial neuron (perceptron).

and receptive lines called dendrites (see Figure 2.2). Learning may be based on two mechanisms: the creation of new connection and the modification of current connections. Each neuron has an activation level which, in contrast to Boolean logic, ranges between some minimum and maximum level.

In artificial neural networks the inputs of the neuron are combined in a linear way with different weights. The result of this combination is then loaded up a non-linear activation unit (activation function), which can in its simplest form be a threshold unit (see Figure 2.2).

Neural networks are often used to widen out, improve and optimize the fuzzy logic based systems by giving them a learning cognition. This learning ability is achieved by presenting a training set of different examples to the network and using learning algorithm which changes the weights (or the parameters of activation function) in such a way that the network will reproduce a correct output with the correct input values. The difficulty is how to guarantee generalization and to determine when the network is sufficiently trained.

Neural networks offer nonlinearity, input-output mapping, adaptivity and fault tolerance [Haykin, 1994]. Nonlinearity is a demanded property if the generator of input signal is inherently nonlinear. The high connectivity of the network ensures that the influence of errors in a few terms will be minor, which ideally gives a high fault tolerance.

2.3 Probabilistic reasoning

As a fuzzy set theory, the probability theory deals with the uncertainty, but usually the type of uncertainty is different. Stochastic uncertainty deals with the uncertainty toward the occurrence of a certain event and this uncertainty is quantified by a degree of probability. Probability statements can be combined with other statements using stochastic methods. Most known is the Bayesian calculus of conditional probability. Probabilistic reasoning includes genetic algorithms, belief networks, chaotic systems and parts of learning theory [Zadeh, 1994].

Genetic algorithms

Genetic algorithms optimize a given function by means of a random search. They are best suited for optimizing the tuning problems in the cases, where a prior information is not available. As an optimization method genetic algorithms are such more effective than a random search.

They create a child generation from parent generation according to a set of rules that mimic the genetic reproduction in biology. Randomness plays an important role, since:

- the parents are selected randomly, but the best parents have greater probability of being selected than the others,
- the number of “genes” to be muted is selected randomly,
- a bit in new child string can be flipped with a small probability.

Probability

Random events are used to model uncertainty and they are measured by probabilities. A random event *E* is defined as a crisp subset of a sample space *U*. The probability of *E* is the proportion of occurrences of *E*.

Subjective probability (Bayesian interpretation of probability) is described as the amount of subjective belief that a certain event may occur and it is the most similar concept to fuzzy logic. The value of 1 is used to denote complete certainty that an event will occur and the value of 0 to denote complete certainty that the event will not occur. The values between 1 and 0 represent degrees of belief. This view differs from the more frequent views of probability. Naturally some authors, like Kosko in [Kosko, 1990] and Kauffman in [Kauffman, 1975], have proposed that the subjective probability theory is the subset of fuzzy logic.

Belief network

A belief network may also be referred to as a Bayesian network or as a causal network. Bayesian network is a model for representing uncertainty in our knowledge and it uses probability theory as a tool to manage this uncertainty. Network has a structure which reflects causal relationships and a probability part which reflects the strengths of these relationships. Users of the network must provide observation based informatics about the value of a random variable. The network then gives the probability of another random variable.

Chapter 3

The principle of fuzzy logic

3.1 Introduction

Fuzzy logic is all about the relative importance of precision. How important is to have exact correct facts? Fuzzy logic is an area of research, which compares significance and precision (see Figure 3.1). Human beings have been doing this for a very long time. In this sense, fuzzy logic is both old and new, because the concept of fuzzy logic relies on the age-old skills of human reasoning.

Fuzzy logic is an easy tool, which relates the mapping input space to an output space. The surveying of input to output is the point of origin for everything. Consider the following examples [Matlab manual]:

- with information about how good was your service at a restaurant, a fuzzy logic system can tell you the amount of money one should tip,
- with your specification of how hot you want the water, a fuzzy logic system can adjust the valve to the right setting,
- with information about how far away the subject of your photograph is, a fuzzy logic system can focus the lens for you,
- with information about how fast the car is going and how hard the motor is working, a fuzzy logic system can shift gears for you.

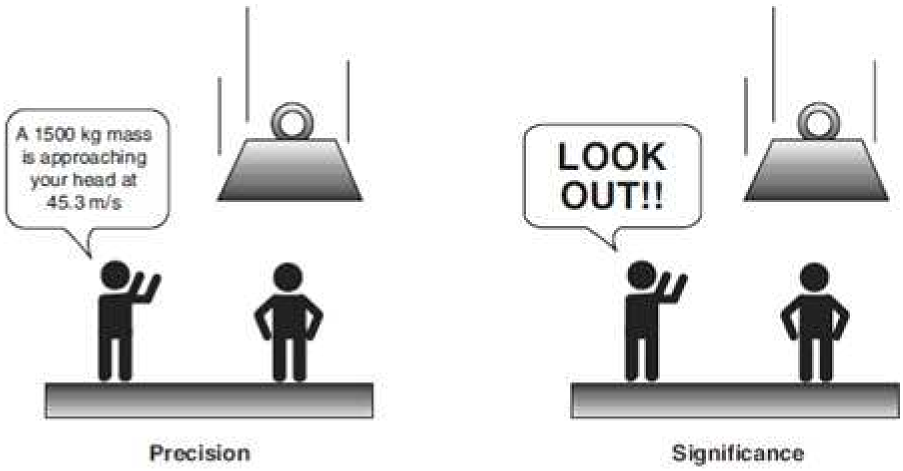


Figure 3.1: Precision and significance in the Real World [Matlab manual].

3.2 Definition of fuzzy set

One of the solving problems of the true description of reality is establishment of multi-valued logic. We do not declare about a given element if it belongs to the set or not, but some "new" expressions are allowed to be used, like "rather belongs to", "rather does not belong to" and conformable. Characteristic function, which determines the element membership of set, takes the values, for example, {0, 0.25, 0.5, 0.75, 1}. If we go further more, we can state the range, and possibly, the whole interval ⟨0, 1⟩. Now an occasion arises to define mathematically the vague terms from common language, like "almost", "nearly", "mostly", etc. Just like that the fuzzy sets differ from classical sets. The characteristic function (so-called membership function) of fuzzy set *A* can take the values inside the true range. Typical example of this function is:

$$\mu_A(x) : x \to \langle 0{,}1 \rangle$$

where $\mu_A(x)$ is a grade (degree) of membership of *x* in set *A*. From the definition it is seen that the fuzzy set theory is a generalized set theory that includes the classical set theory as a special case. Membership function can be also viewed as a distribution of truth of a variable.

The domain of a function of values, where the membership function is defined, is known as universum. The graphical representation is shown in Figure 3.2.

Another way how to describe the fuzzy set is cuts and levels [Navara & Olešák, 2007]. Roughly speaking, α-cut of fuzzy set *A* is an interval or group of intervals, where the value

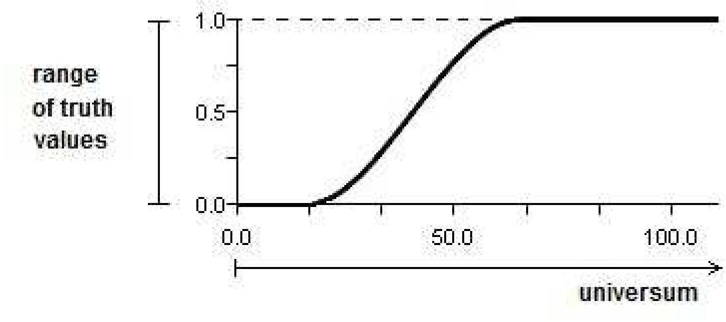


Figure 3.2: Example of membership function of fuzzy set.

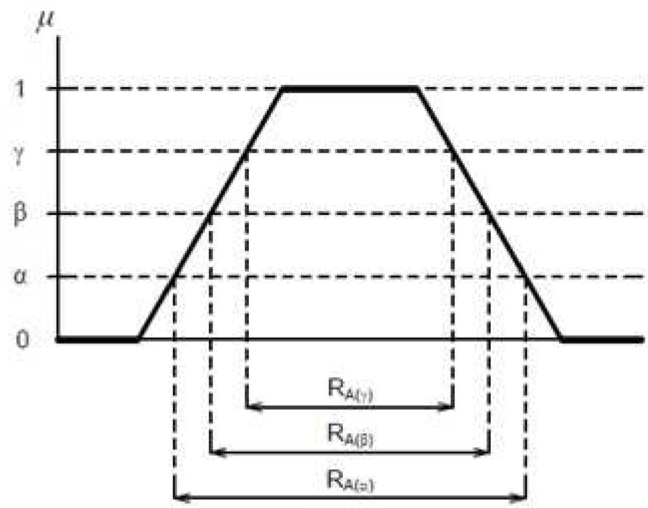


Figure 3.3: Cuts of fuzzy sets for α-, β-, γ-level.

of the membership function is more than or equal to the value of α. Mathematically written:

$$R_A(\alpha) = \{x \in X : \mu_A(x) \geq \alpha\}$$

A whole fuzzy set can be written with systems of cuts, where α is from the whole true range (see Figure 3.3).

To understand the fuzzy sets is the need to realize the difference between vagueness of information used in fuzzy description of the world and classical probability. The difference is mainly in the original uncertainty. In using the fuzzy approach very the description of the phenomena is already indefinite and remains indefinite throughout the research. While the probability takes into account the uncertainty before close examination of reality, but it is no longer any uncertainty after the investigation. The resulting answer is unequivocal.

Fuzziness is a type of imprecision, which characterizes classes that cannot have or have not gotten for various reasons sharply defined boundaries [Zhang & Goodchild, 2002]. These inexactly defined classes are called *fuzzy sets*. Fuzziness is often epiphenomenon of complexity. It is suitable to use fuzzy sets whenever one is concerned with ambiguity, vagueness and ambivalence in mathematical or conceptual models of empirical phenomena. Fuzzy set theory is a generalization and not a replacement for the better-known abstract set theory which is often referred to as Boolean (or classical) logic.

Fuzziness is not a probabilistic attribute, in which the degree of membership of a set is linked to a given statistically defined probability function. It is rather an acceptance of possibility that an element is a member of a set or that a given statement is true. The examination of the possibility could be based on subjective, intuitive knowledge or preferences, but it can also be referred to clearly defined uncertainties that have a basis in the probability theory.

3.3 Matching the Boolean logic and fuzzy logic

Conventional or crisp sets allow only binary membership functions (true or false). Fuzzy sets admit the possibility of partial membership, so they are generalizations of crisp sets to situations where the class boundaries are not, or cannot be sharply defined. The same is true for an environmental property such as “internal soil drainage”, which comprises all conditions from total impermeability to free draining.

A *crisp set* is a set in which all members match the class concept and the class boundaries are sharp. The degree to which an individual observation *z* is a member of the set is expressed by the membership function *F*, which can take the value 0 or 1 for Boolean sets. Note that *z* is used here as a general attribute value. Formally:

$$\mu^B(x) = 1 \qquad \text{if } b_1 \le x \le b_2$$
$$\mu^B(x) = 0 \qquad \text{if } x < b_1 \text{ or } x > b_2$$

where b_1 and b_2 define the interval of set *A*.

A *fuzzy set* is defined mathematically as follows:

$$A = \left(x, \mu_A^F(x)\right) \qquad \text{for all } x \in Z$$

If *Z* denotes a space of objects, then the fuzzy set *A* in *Z* is the set of ordered pairs, where the membership function μ_A^F is a number in the range 0.1 with 0 representing non-membership of the set and 1 representing full membership of the set. The grade of membership of *x* in set *A*

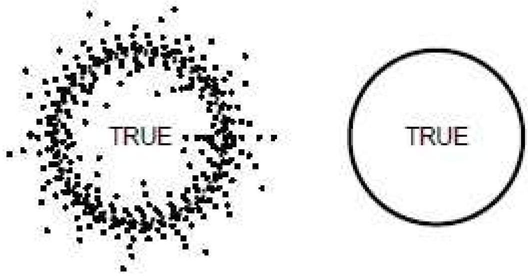


Figure 3.4: Boundaries of fuzzy set (left) and Boolean set (right).

reflect a kind of ordering that is based on admitted possibility not on probability. The value of membership function of object *x* in set *A* can be interpreted as the degree of compatibility of the proposition, which is associated with set *A* and object *x*. Simply said, the membership function of *x* in set *A* specifies the extent to which *x* can be considered as a component of set *A*. The value of the membership function indicates, to what degree is a sighting *x* a member of set *A*.

In Figure 3.4, there is illustrated the difference between Boolean (crisp) and fuzzy sets.

In fuzzy sets, the grade of membership is reflected in terms of a scale that can fluctuate continuously between 0 and 1. Individuals, which are closed to the core concept, have values of the membership function close to or equal to 1, individuals further away have smaller values. This immediately gets around the principle problems, but the middle is excluded, and the individuals can be members of more than one set (to different degrees). The problem is to specify the membership function clearly, unambiguously.

The boundary values of crisp sets are in many cases picked on the basis of expertise (e.g. boundary values of demanding criteria chosen by custom or law) or by using methods of numerical taxonomy. Usually the stored classes based on expert knowledge are set up without direct reference to the local data set. Thus they might approximate distribution, but in any statistical sense they are not optimal, there require only boundary values (the lower and the upper).

The “natural” classification methods make classifications, which are optimized locally to confront the data set. For all purposes, the choice of classification method, values of parameters, etc. may expressively impress the results of the classification. Methods of numerical taxonomy are mainly tools for researching, which are data driven. These options are also acceptable for fuzzy sets.

The simpler approach uses a membership function with which elements can be associated to a membership grade. This is known as the Semantic Import Approach or Model. The second approach is analogues to cluster analysis and numerical taxonomy, where the value of membership function is a function of the classifier, which is used. This model is known as the method of fuzzy clustering (*c*-means). To draw the comparison to the crisp clustering, in fuzzy clustering the element of a set can pertain to several of created clusters to different degrees. The semantic import is the second method for assessment of the membership functions, which is based on the expert knowledge.

3.4 The membership function

The fuzzy membership function is a basis for proposing the fuzzy systems. Not only fuzzy rules, but also the construction and generating acceptable membership function, have been solved for some decades. Even though there have been many efforts, which were focused on automatically generating fuzzy rules and optimization techniques, the automatic generating of membership function is still based on expert knowledge.

3.4.1 The semantic import approach

The semantic import approach is useful in those situations, when users have a very good and qualitative notion of grouping the data, but because of several reasons they have difficulties with the exactness, which is connected to the standard Boolean model. The choice of boundaries for crisp sets and of intervals could be objective of subjective procedure in dependence on the method of defining the classes. Very often the thought is related to the selection of acceptable boundaries between intervals of classes. The same is valid for assigning the membership function of a fuzzy set. The membership function should provide the grade of membership at the centre of set equal to 1.0 that in an appropriate way it falls off through the fuzzy boundaries to the area outside the set, where it takes value 0. The place, where the grade of membership is 0.5, is called the cross-over point. The membership function must be defined in such a manner that these conditions are obtained, so not all functions are possible.

As there are different types of probability distribution, there can be different types of fuzzy membership functions, which have been already proposed (see Figure 3.5), for example trapezoidal, triangular, Gaussian or bell-shaped. Each of them is more appropirate for different datasets. They might be symmetric or consisted of two different functions, sharp or having a certain part completely compatible with the class. Choosing the type of the function has been considered as a subjective procedure [Lee, 1990]. Regardless of its type, the membership function can be described with three parameters: support, boundary and core or prototype, as shown in Figure 3.6. The most common membership functions are the linear, composed of lines, or “sinusoidal“ like Gaussian or bell-shaped functions, consisting of curves. Likewise, tthe polynomial functions are also defined.

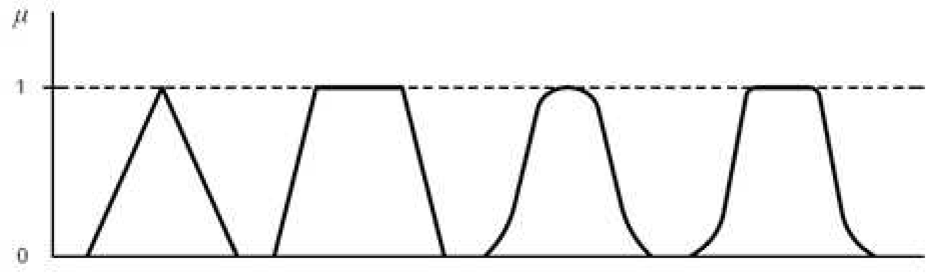


Figure 3.5: Example of possible shapes of membership function.

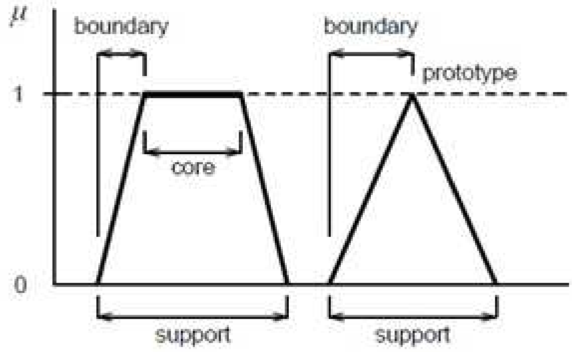


Figure 3.6: Parameters of membership function in two types of membership functions: trapezoidal and triangular.

The linear fuzzy membership function is given by a pair of sloping lines that peak at the membership function equal to a value 1 for the central concept of the set and have membership function values equal to a value of 0.5 value at the boundaries. The slope of the line gives the width of the fuzzy transition zone. The areas inside the sloping lines, but outside the Boolean rectangle are zones of partial truth.

The sinusoidal membership function is given by the following formula:

$$\mu_A^F(z) = \frac{1}{(1 + a(z-c)^2)} \qquad \text{for } 0 \le z \le P$$

where *A* is the set in demand, *a* is a parameter is a parameter regulating the shape of the function and *c* defines the value of the property *z* at the central concept (see Figure 3.7). The varying the value of *a*, the form of the membership function and the position of the cross-over point can be fitted to Boolean sets of any width. The value of *z* flanking the central concept from the cross-over points can be thought of as the transition zones surrounding the central concept of the fuzzy set.

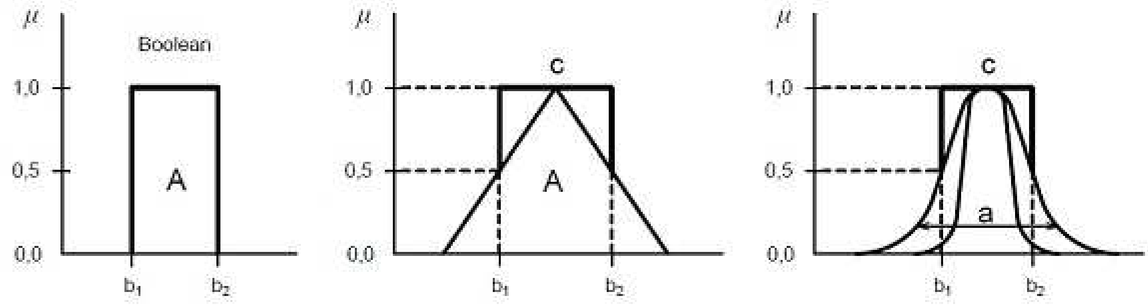


Figure 3.7: Boolean and fuzzy membership function for the semantic import method.

In many situations only the lower or upper boundary of a class may have practical importance. This could be true in situations, for example, where it is wished to know the depth of soil enough for a given purpose.

In contrast to the simple crisp set where membership values are directly 0 or 1, the semantic approach transforms the data to a continuous membership function fluctuating from 0 to 1. The value of membership function gives the degree to which the elements belong to the set in question. Fuzzy classification is in this way able to sort out the problem of unrealistic sharp class boundaries in a simple and intuitive way with the help of defining the class limits and their dispersion indices.

Compared with Boolean classification, where only the boundary values have to be chosen, the extra problems of the semantic import approach are choosing the shape of function for establishing class membership and selecting the values of the dispersion indices. For that reason the semantic import approach needs more information than the classic Boolean method, but the pay off is extra sensitivity in data analysis.

3.4.2 Fuzzy clustering

Even though the semantic import approach is extremely flexible for creating the fuzzy classes, it can never be optimal. Very often it is not possible to know which classification is useful or suitable and the searching techniques that can suggest appropriate overlapping classes. Many authors prefer to call this method continuous classification rather than fuzzy classification, because of the continuity of the classes in attribute space.

With the increasing number of variables the number of membership function for each variable rises. There is, in general, an increase in the number of rules as well. One of the challenges confronting the use of fuzzy logic is the elimination of excessive rules. One way to decrease the number of rules is demonstrated with the use of fuzzy union among anterior variables. Another way to hold the number of membership function down is to search for clusters in the data.

Clustering comprehends a number of mathematical techniques for identifying natural groupings in a data set.

The main purpose of unpursued classification, alias clustering, of a set of objects is to reveal the natural clusters found in the similarities and the differences between a pair of entities. Many various algorithms have been proposed which depend on the definition or the interpretation of natural clusters.

The goal is to establish data points to clusters in the way that two feature vectors from the same cluster are as similar as possible and two feature vectors from other clusters are as dissimilar as possible. The purpose of fuzzy clustering methods is to separate out a given dataset into a set of clusters based on similarity. In classical cluster analysis the particular data must be associated to just one cluster. Fuzzy cluster analysis releases this requirement by the smooth membership degrees. In this way it is possible to dispose of data, which belong to more than one cluster at about the same time.

The most widely known method of fuzzy clustering is *the fuzzy c-mean* method [sometimes referred to as FCM method], originally proposed by Dunn [Dunn, 1973] and generalized by Bezdek in 1981 [Bezdek, 1981, Bezdek & Hathaway, 1988] and other authors [Gustafson & Kessel, 1979, Klawonn & Keller, 1999, Höppner et al., 1999] as an improvement on earlier clustering methods. The method has strong relations with conventional methods of numerical taxonomy. Data reduction is realized by translating a multiple attribute description of an object into c membership values to c clusters.

Usually, the membership functions are defined on the base of function of distance, where the membership degrees represent closeness of elements to cluster centres (i.e. prototypes). The placing of these centroids is best when they are as far away from each other as possible. With the choice of a suitable distance function, the different cluster shapes can be identified. Nevertheless, these approaches are very difficult to describe specifically, how the fuzzy cluster structure relates to the data, from which it originates from.

The idea of fuzzy c-mean algorithm is using the weights that minimize the total weighted mean-square of error. The fuzzy c-mean algorithm attempts to divise a finite set of elements into a collection of c fuzzy clusters with relation to some given criterion. For given a finite dataset, the list of c cluster centres is returned by the algorithm. The list of c cluster centre proclaimed the belonging of each element to the cluster.

The clusters are optimal in the sense that the multi-variate within the cluster variance is minimal. Near-zero variance means that all objects have nearly equal attributes, which means a high density and small distances between them in an attribute space [Sato et al., 1997]. On the contrary, large variance is equivalent to low density and large distances in the attribute space.

While the boundaries between clusters in attribute space should be situated in the lowest density zones, an optimal clustering process should be considered equal to the dense places in attribute space as cluster centre.

Fuzzy *c*-means works by a repetitive procedure, which usually begins with initial random apportionment of the objects to be classified to *c* clusters [Kosko, 1994]. With regards to the allocation of clusters, the centre of each cluster is calculated as the average of the attributes of the objects. Afterwards, the objects are redistributed among the clusters according to the relative likeness between objects and clusters. The similarity index is ordinarily a well-known distance measure. The redistribution is in process by iteration until it is achieved to a stationary solution, where similar elements are gathered in one cluster.

Redistribution of elements in traditional crisp *c*-means is always to the nearest cluster. With the membership function is equal to a value of 1, the element is allocated to this cluster and with membership function equals to a value of 0, the element is allocated to any other cluster. With fuzzy *c*-means, the membership values may range between values 0 and 1. In the normal *c*-means, the *c* memberships of each object amount to 1.

The pure result of fuzzy *c*-mean clustering is, that each individual multivariate objects (points, lines, polygons) are associated to a value of membership function with due regard for each of *c* overlapping clusters. The centroid of each cluster is chosen optimally with a view of the data. In the difference of the technique, the values of membership function sum to 1 more likely than separately as in case with the semantic import approach. That means, the sets are divided according to the importance rather than all sets having equal value, as with the semantic approach.

3.4.3 Usage of semantic import approach and fuzzy clustering

The decision to use semantic import approach or fuzzy clustering (especially *c*-means) approach in continuous classification depends on the context of the problem and also on the level of previous information. In situations where obvious, specific and functional classification system exists, the primary objective semantic import approach offers great advantages but the precise definition of the fuzzy boundaries demands some special thought. Studies informed on have shown that the semantic import continuous classes are more robust and less predisposed to errors and extremes than simple Boolean classes that use the same attribute membership functions in not only data space.

The fuzzy *c*-means approach, the criteria that differentiate from the ultimate classes are a result of the analysis rather than input to the model. These criteria could be a collection of non-linear functions of the original attributes; consequently, there is no perspicuous relation between membership values and attribute values. While elements, which have coincident attribute values, will finish up with identical memberships, in general the contrary is not true because of the mutual equalization possible among attributes. Forasmuch as the reduction of data is one of the main reasons for using class membership instead of the original values of attributes, some loss of information will always be confronted.

3.5 Fuzzy operations

Operations on fuzzy sets are, by nature, based on operations on membership function [Zadeh, 1965]. The fuzzy logic is founded on mathematical framework, which uses an extension of the classical Boolean logic, but we operate also with values between 0 and 1. Operations on fuzzy sets are substantially much more varied than operations on classical sets. In reality, most operations on fuzzy sets do not correspond to any on the classical set theory. The following are five types of operations on fuzzy sets [Bělohlávek et al., 2002]:

- modifiers
- complements
- intersections
- union
- averaging operations

The modifiers and the complements operate on one fuzzy set. The intersections and the unions operate on two fuzzy sets, but their application can be expanded to any number of fuzzy sets by means of their capability of associativity. The averaging operations, which are not associative, generally, operate on more than or equal to two fuzzy sets.

3.5.1 Modifiers

Modifiers are operations of one-component whose basic purpose is to modify fuzzy sets to account for linguistic limitations, such as *more or less, very, fairly, rather, extremely, moderately*, etc., in representing expressions of natural language. Each modifier, *m*, is an ascending and usually continuous one-to-one function of the form

$$m : \langle 0,1 \rangle \rightarrow \langle 0,1 \rangle$$

which classifies to each membership grade $A(x)$ of a given set *A* a modified grade $m(A(x))$. The modified grades for all $x \in X$ define a new, modified fuzzy set.

Mention that function *m* is absolutely independent of elements *x* to which values $A(x)$ are specified. It depends only on the values themselves. In describing its formal properties, in that way, we may ignore *x* and suppose that the argument of *m* is an arbitrary number *a* in the interval $\langle 0,1 \rangle$.

Generally, a modifier preserves the sequence even if it increases or decreases values of the membership functions to which it is applied. Modifiers are essentially of three types, depending on which values of the membership functions they increase or decrease; modifiers that increase all values; modifiers that decrease all values; modifiers that increase some values and decrease other values.

As is well known, the modifiers do not always properly capture the meaning of linguistic hedges in natural language.

3.5.2 Complements

Similarly to modifiers, complements of fuzzy sets may be defined by means of acceptable unary operations on $\langle 0,1 \rangle$. While modifiers preserve the sequence of membership degrees, complements overturn the sequence. Exceedingly, each fuzzy complement must meet the following two requirements, at least:

- $c(0) = 1$ and $c(1) = 0$
- for all $a, b \in \langle 0,1 \rangle$ if $a \le b$,then $c(a) \ge c(b)$.

The first requirement guarantees that all fuzzy complements collapse to the unique classical complement for classical (crisp) sets. The second requirement guarantees that increases in the membership degree in A set do not result in increases in the membership degree in the complement of A set. This is substantial since any increase in the degree of the membership of an element in a fuzzy set cannot increase concurrently the degree of nonmembership of the same element in the same fuzzy set.

The operation of complementation is defined in terms of:

$$\mu_{\neg A}(x) = 1 - \mu_A(x)$$

Even though the previous requirements are adequate to characterize the largest class of acceptable fuzzy complements, two additional requirements are inflicted on fuzzy complements by most applications of fuzzy set theory:

- *c* is a continuous function

- $c(c(a)) = a$ for all $a \in \langle 0,1 \rangle$

These requirements guarantee that infinitely, small changes in the argument do not implicate discontinuous changes in the function, and that fuzzy sets are not changed by double complementation.

For determination the most suitable complement in terms of each particular application is a problem of knowledge retrieval, somewhat similar to the problem of constructing the membership functions. With the regard to a class of fuzzy complements, the constructing problem reduces to the problem of determining the right value of the significant parameter.

3.5.3 Intersections and unions

Intersections and unions of fuzzy sets, alternatively designated as *i* and *u*, are generalizations of the classical operations of intersections and unions of crisp sets. They may be defined through the use of appropriate functions that represent each pair of real numbers from the interval $\langle 0,1 \rangle$, which represents the degrees $A(x)$ and $B(x)$ of given fuzzy sets *A* and *B* from some $x \in X$) into a single number in the interval $\langle 0,1 \rangle$, which represent the membership degree $(A \cap B)(x)$ of the intersection of *A* and *B* or membership degree of the union of *A* and *B* for the given *x*, for all $x \in X$, consequently:

$$C(x) = (A \cap B)(x) = i(A(x), B(x))$$

and

$$D(x) = (A \cup B)(x) = u(A(x), B(x))$$

The membership function $\mu_C(x)$ of intersection is defined by:

$$\mu_C(x, y) = \min\{\mu_A(x), \mu_B(y)\}$$

The membership function $\mu_D(x)$ of intersection is defined by:

$$\mu_D(x, y) = \max\{\mu_A(x), \mu_B(y)\}$$

In contradistinctions to their classical counterparts, fuzzy intersections and fuzzy unions are not unique. It is a natural entailment of the well-established fact, that the linguistic expressions "*x* is a member of *A* and *B* sets" and "*x* is a member of *A* or *B*" have different connections.

It is necessary to characterize the classes of fuzzy intersections and fuzzy unions as widely as possible, to be able to capture the different meanings.

It has been introduced that the operations known in the literature as triangular norms (or t-norms) and triangular conorms (or t-conorms), have exactly those properties, that are necessary, on intuitive grounds, for fuzzy intersections and fuzzy unions, possibly. Different t-norms and t-conorms were proposed for special purposes, each varying the gain of the operation. The class of t-norms (fuzzy intersections) is characterized by four requirements; the class of t-conorms (fuzzy unions) is also characterized by four requirements, three of which are identical with the requirements for t-norms. In the following list, the requirements for t-norms *i* are paired with their counterparts for t-conorms *u*, and must be satisfied for all $a, b, d \in \langle 0,1 \rangle$:

- $i(a,1) = a$ (boundary requirement for *i*)
- $u(a,0) = a$ (boundary requirement for *u*)
- $b \le d$ implies $i(a,b) \le i(a,d)$
- $b \le d$ implies $u(a,b) \le u(a,d)$ (monotonicity)
- $i(a,b) = i(b,a)$
- $u(a,b) = u(b,a)$ (commutativity)
- $i(a, i(b,d)) = i(i(a,b), d)$
- $u(a, u(b,d)) = u(u(a,b), d)$ (associativity)

It is easy to see, that fuzzy intersections collapse to the classical set intersection when it is applied to crisp sets: $i(0,1) = 0$ and $i(1,1) = 1$ follow directly from the boundary requirement; $i(1,0) = 0$ and $i(0,0) = 0$ follow then from commutativity and monotonicity, respectively. Likewise, the fuzzy unions collapse to the classical set union when applied to crisp sets. Commutativity requirements ensure, that fuzzy intersections and fuzzy unions are symmetric operations, indifferent to the order in which sets to be combined are contemplated. Together with monotonicity requirements, they guarantee that fuzzy intersections and fuzzy unions do not decrease, when any of their arguments are increased, and conversely, do not increase when any arguments are decreased. Associativity requirements allow expanding fuzzy intersections and unions to more than two sets.

Whatsoever combination of fuzzy counterparts of the tree classical set theoretical operations (complement, intersection, union) will be chosen, some properties of the classical operations, properties of the underlying Boolean algebra, are necessarily contravened. This is a consequence of imprecise boundaries of fuzzy sets. The standard fuzzy operations contravene only the law of impacted middle and the law of contradiction. Some other combinations keep these laws, but contravene distributivity and idempotence [Klir & Yuan, 1995].

3.5.4 Averaging operations

Fuzzy intersections (t-norms) and fuzzy unions (t-conorms) are special types of operations for grouping fuzzy sets. Existing two or more fuzzy sets, they produce a single fuzzy set, a complex of the existing sets. While they do not overlay all grouping operations, they overlay all grouping operations that are associative. The residuary grouping operations must be defined, because of deficiency of associativity, as functions of n arguments for each $n \geq 2$. These residuary grouping operations are called *averaging operations*. As the name prompts, they average in various ways of membership functions of two or more fuzzy sets defined on the same universal set. They do not have any counterparts in Boolean set theory. In fact, an average of several characteristic functions of classical sets is not a characteristic function, in general. Nevertheless, classical sets can be averaged if they are viewed as special fuzzy sets.

There exist some requirements for any averaging operations with n arguments $(n \geq 2)$. An average of equal numbers must result in the same number (idempotency); the average does not decrease when any of the arguments increase (monotonicity). In addition to these essential and easily understood requirements, averaging operations on a fuzzy set are usually expected to meet two additional requirements. The average is a continuous function; and the average is a symmetric function in all its arguments. These additional requirements guarantee that small changes in any of the arguments do not result in discontinuous changes in average and they capture the usual supposition that the grouped fuzzy sets are equally important. If the supposition is not reserved in some application contexts, the symmetry requirement must be relaxed.

3.5.5 Fuzzy numbers

Fuzzy numbers are special fuzzy sets in set of real numbers $R = (-\infty, +\infty)$. In principle it is assumed, that fuzzy numbers have special shape, which is visible in Figure 3.8. Any fuzzy interval *A*, for which $A(x) = 1$ for exactly one $x \in R$, is called a fuzzy number [Novák, 2000].

Fuzzy number intuitively represents the value, which is inaccurate, that means the value, which can be verbally characterized as “about z_0“, “roughly z_0“, etc. Typical examples are “about 5”, “roughly 1200”, “approximately 1 m”, etc. These numbers are not exceptional, more to the contrary. In practise, people work almost exclusively with numbers, which are fuzzy. Only extraordinarily the numbers are meant as accurate, precise numbers.

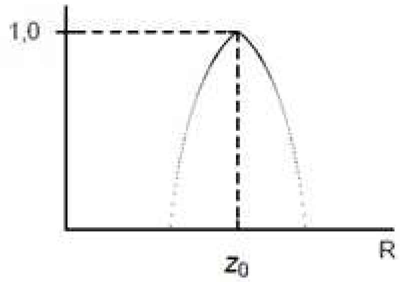


Figure 3.8: The membership function of fuzzy number " about z_0".

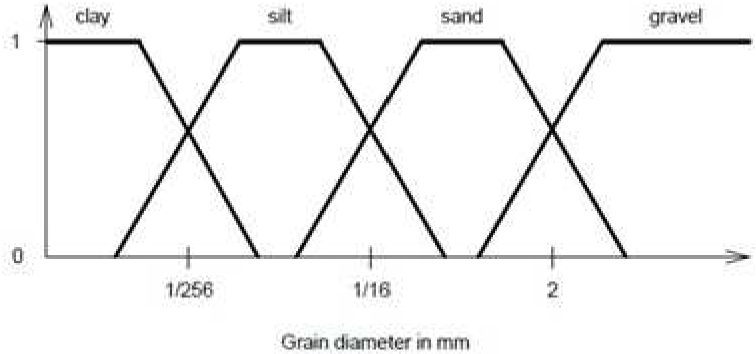


Figure 3.9: Representation of the geologic variable "grain size" [Klir, 2004].

Clearly, in contrast to Figure 3.5, which represents some variant type of fuzzy numbers, the Figure 3.9 shows fuzzy intervals. Every fuzzy interval *A* may ordinarily be expressed for all $x \in R$ in form:

$$A(x) = \begin{cases} f_A(x) & \text{when } x \in \langle a, b) \\ 1 & \text{when } x \in \langle b, c) \\ g_A(x) & \text{when } x \in (c, d\rangle \\ 0 & \text{otherwise} \end{cases}$$

where *a, b, c, d* are specific real numbers such that $a \le b \le c \le d$, f_A is a real-valued function which is increasing and right continuous, and g_A is a real-valued function which is decreasing and left continuous.

The construction of fuzzy model is based in reality on the linguistic variables and their values, which have the specific meaning in the real world. For the universe of a variable, two appropriate primitive terms are reciprocally antonyms [Matlab manual]. Do not forget, that negation in fuzzy logic does not mean the same as antonyms, but rather, its counterpart. It is necessary to distinguish antonyms from negation and be careful with using these expressions. For example the variable

Height, terms *tall* and *small* are expressing the opposites, but the negation to *small* – *not small* does not equal to *tall* and contraversially *small* $\neq$ *not tall*.

Using more general expressions, for example *not very tall*, is a much more complicated situation that the previous example. In this case the operator *very* is negated, or the entire expression. The first case is more ordinary. However, with the theory of negation the operator is not already processed.

Further more, when using the modifiers the variables can be changed. The modifiers are adverbs, as *very*, *fairly*, *quite*, *more or less*, etc. One value represents the neutral case and other modifiers should be used symmetrically with the respect to the neutral value. In the example, the variable *Height* can be evaluated by terms: *very tall*, *tall*, *fairly tall*, *more or less tall*, *not tall*, *medium*, *not small*, *more or less small*, *fairly small*, *small*, *very small*. The reality can be constructed as compound expressions with using connectives as *and* and *or*. The linguistic structure is capable to reflect the reality better.

The property of shape invariance, which holds precisely for addition and subtraction and somewhat for other operations, makes it possible to represent fuzzy numbers in parametric form and thus translate various operations on fuzzy numbers into corresponding arithmetic operations on their parameters [Yager & Filev, 1994]. This idea has the potential for many significant applications to the approximate analysis of both fuzzy and nonfuzzy system.

It was described the relation between the linguistic meaning of fuzzy terms and mathematical fuzzy model. The natural linguistic rules can be involved and adapted on basics of defined fuzzy models with mathematical methods.

Chapter 4

Expert system

4.1 From reality to fuzzy system

Information about the structure of the world and its patterns are represented by expressions such as “acceleration is small and positive”. The quantity “acceleration”, which affects to the concerns, are called linguistic variable. It can take the certain values “small and positive”. These linguistic values correspond to the appropriate fuzzy sets and its membership functions. Each expression belongs to some account. The transform, which assign each expression its meaning is called semantics of the language.

The linguistic variable can be defined as a variable, whose values are expressions of some language as a real numbers within a specific interval of real numbers. Examples of geological variables are: distance from source, depth or grain size. Each linguistic variable consists of:

- the name of linguistic variables, which should reflect the meaning of the base variable involved,
- the set of terms (linguistic values, corresponding to fuzzy sets),
- the universe,
- the syntactic rule (a generative grammar),
- the semantic rule (degree of agreement of terms with its meaning).

The terms can be combined into a compound term or modifies in meaning using language operators. For example “Speed is a large and a small distance” or with using language variables “Speed is about big”. Generative grammar generates terms (the shape and position of the membership function). If all terms contained in language can be generated like this, then we call them structured.

Another important question is the meaning of compound expressions. This evidently depends on the importance of each elementary terms from which it is composed. The very composition is realized by using the connectives as *and*, *or*, etc. Therefore, the operations can be used with fuzzy sets.

4.2 Expert system

Conventional programs deal with the data in sensor-based way. The correct view of the task is fundamental and required, and global flow has to be proposed as well as algorithmic details. The programme debugging is often time-consuming, and its next modification is difficult. [Zimmermann, 2001] In support of activities requiring significant human expertise, these methods have been found unsatisfactory.

An expert system is a set of programmes that use encoded knowledge to solve problems in a specialized field that normally demands human expertise.

Basically, the expert systems can be knowledge-based, model-based or hybrid. In *knowledge-based systems*, the relationship between input and output linguistic variables is described by collections of if-then rules. These rules try to catch the knowledge of a human expert, often expressed in natural language. The *model-based systems* are based on traditional systems modelling, but they apply suitable areas of fuzzy mathematics. The *hybrid systems* are combinations of knowledge-based and model-based fuzzy systems. Presently, knowledge-based systems are more developed than model-based or hybrid systems.

The two main components of a knowledge-based expert system are a collection of rules, called the knowledge base, and a way of reasoning, called the inference engine. Methods of their construction, and therefore construction of expert systems, are subject of knowledge engineering.

The knowledge engineer does not need to have to have knowledge in any expert domain, but he does need to know, how to transform these expertise into the rules, that the system will use [Zimmermann, 2001]. The parts of the expert system, that do not contain domain specific informations, are contained within the expert system unit. This unit is a general toolkit that can be

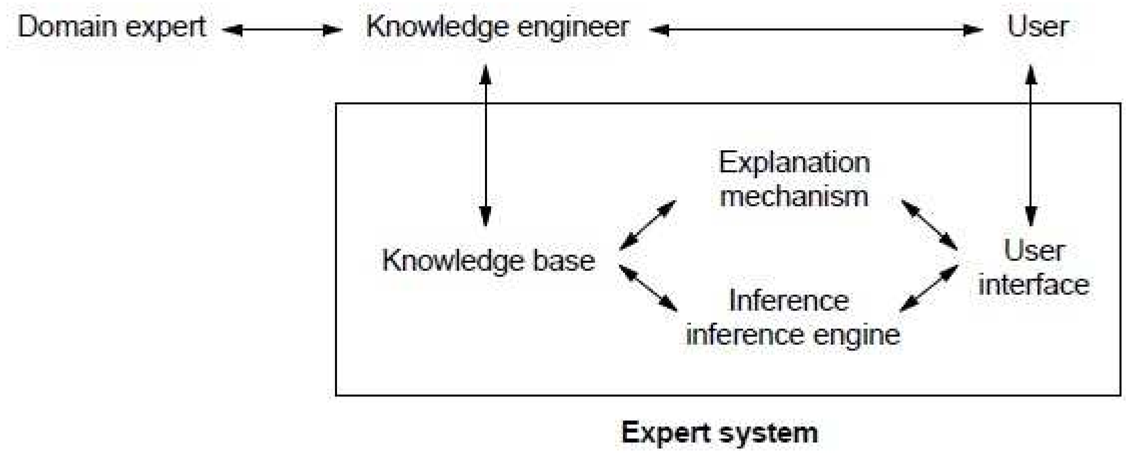


Figure 4.1: Architecture of an expert system.

used for creating of different expert systems. The knowledge engineering is an important component of the development of any expert system (Figure 4.1).

One potential problem with the expert system is the number of comparisons, which need to be made between rules and facts in the database. In some events, when there are hundreds of rules, the comparisons between each rule can be unpractical.

The fuzzy logic controller and expert systems may be considered as a special rule based systems that use the fuzzy logic. As some systems must predefine membership function, fuzzy inference rules to map numeric data into linguistic variable terms.

Expert systems and knowledge-based systems have appeared from research in the area of artificial intelligence, i.e. the impersonation of train of human thought in a computer. This research has conduced to the concept, that human behaviour is constantly being adapted by new knowledge and experience, while the control processes of the brain remain roughly unchanged.

It has been proposed, that an intelligent computer system should contain come software, which would have control thoughts (the inference engine) and other software constituting knowledge (the knowledge base). The computer systems, which were built with this type of software architecture, are known as knowledge-based systems (see Figure 4.2).

The knowledge-based systems can be used in order to realize the specialized human tasks, to impersonate the behaviour of specialists in expert fields. The term expert system was created to describe such systems. In many applications, the flexibility, which is offered by having a separate knowledge base, is in itself satisfactory validity for using the expert system. Many expert systems are pronounced as capable of explaining their reasoning, even if the explanation is many times only a race of the programme execution.

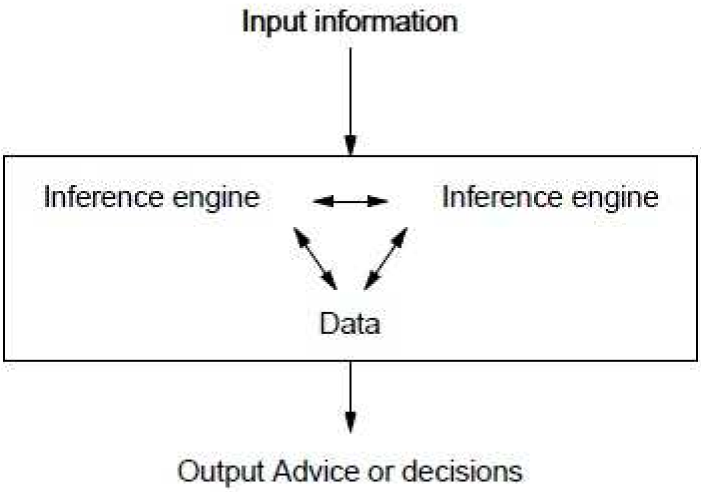


Figure 4.2: Architecture of a knowledge-based system.

The production rules are in most cases used formulation of the knowledge, due to the natural syntax and the resemblance to the ordinary expert's expressions. The production rules are suitable for fuzzy reasoning. The production rules are usually mentioned as simple *if-then* rules.

A fuzzy rule based expert system includes fuzzy rules in its knowledge base and derives conclusions from the user inputs and the fuzzy inference rules to reflect the numeric data into linguistic variable terms (for example very high, young, etc.) and to make fuzzy reasoning work. The linguistic variables are usually defined as fuzzy sets with appropriate membership functions. The rules in fuzzy expert systems usually have the following form:

"If x is low and y is high then z is medium."

Where *x* and *y* are input variables (names for known data values), *z* is an output variable (a name for the data value to be computed), *low* is a membership function (fuzzy subset) defined on *x*, *high* is a membership function defined on *y*, and *medium* is a membership function defined on *z*. The *if*-part of the rule is called the premise or antecedent and expresses the conditions, when the rule should be applied. This is a fuzzy logic expression that describes to what degree the rule is suitable. The part of the rule following the "then" is the rule's conclusion or consequent. *Then*-part, called consequent, is a set of actions to be executed when the rule is applicable. This part of the rule refers a membership function to each of one or more output variables. Most tools for working with fuzzy expert systems allow more than one conclusion per rule. A typical fuzzy expert system has more than one rule. The complete group of rules is collectively known as a rule base or knowledge base.

An inference engine is the mechanism, which corresponds to the case specific data against the knowledge base, makes use of appropriate rules and deduces conclusions. For knowledge presented in production rules, there are two basic strategies of connection between rules, forward chaining and backward chaining. In the former, data-driven method, the rules, for which the antecedents are satisfied, are started up to derive conclusions. The latter is a goal driven method. The rule, whose consequent possesses the fact, is chosen and attestation of its antecedents follows. It is possible to combine both approaches.

An expert system is composed of a friendly user-interface. Use of natural language and problem-oriented terms enables the user to qualify the problem accurately and understand better the conclusions. In this manner the system is conformed to the user. The expert systems can make use of an explanation system to give users the possibility to follow the reasoning of system. Different bases of knowledge, inclusive of a different domain, can be joined to the remaining expert system (see Figure 4.1).

Not all problems are suitable to be solved by the expert systems, in spite of the expert systems are suitable and useful auxiliary to support human logical meaning. There exist some directions, to spare some time and money with the use of the inappropriate problem [Cawsey, 1994], [Waterman, 1986]:

- The task does not require common sense.
- The task does not require cognitive skills.
- The problem is of proper size and scope.
- The problem may be solved using symbolic reasoning techniques.
- The problem cannot be easily solved using more traditional computing methods.
- Genuine experts exist and can articulate their skills.
- The experts agree on solutions.
- Human expertise is not available in all situations where it is needed.
- Need of solution must justify the costs involved in development.

The expert system may be constructed, if the problem was found appropriate. It is necessary to start with the development by small prototype, which focuses on narrow problem area. Another improving can continue, if such prototype proves good.

It is eventful for well working system to archive true and complex information, but this is very difficult task. Therefore the knowledge retrieval is the most difficult part of the development of an expert system.

The main efficiency of the expert systems consists in their knowledge bases. There exist two main accesses to the rule generation, empirical data – driven clustering methods and knowledge acquisition of domain experts. It is possible to combine both of these accesses [Niskanen, 2004]. Nevertheless, it is very slow and protracted process to document the knowledges from the minds of experts. If the inference mechanisms are defined for fuzzy rules, it is auspicious to take advantages of linguistic models in knowledge engineering.

In comparison with the traditional programming code, fuzzy rules are as comments in the code. The modelling of fuzzy systems is made very simple and easy to pursue with these notation without using any mathematical formulas. In terms of applicability it seems to be better than appropriate conventional reasoning systems, although the fuzzy reasoning is often based on the intuitive and subjective suppositions [Niskanen, 2004].

4.3 Fuzzy inference system

Fuzzy inference is the process of expressing the setting up of characteristics from a given input to an output by means of the fuzzy logic. The setting up renders the basis, from which decisions can be made or from which patterns can be resolved. The procedure of fuzzy inference includes all parts, which have been already described in Chapter 3: *membership functions*, *fuzzy operations*, *if-then rules*.

There are two basic types of fuzzy inference systems, which can be implemented: *Mamdani-Assilian* and *Takagi-Sugeno type*. These two types of inference systems differ slightly in the way, how the outputs are determined.

The fuzzy inference systems have been applied in branches as data classification, decision analysis, expert systems, automatic control and computer vision. [Matlab manual] Because the fuzzy inference systems have multi-disciplinary character, fuzzy inference systems are identified with lot of names, like fuzzy rule based systems, fuzzy expert systems, fuzzy modelling, fuzzy logic controllers and last but not least fuzzy systems.

During the development of expert systems, communication with experts and users is founded on natural language. Of course it means the natural language in the problem domain. In an additional manner, the knowledge is often inaccurate and the knowledge base is consisted of rules neither totally certain nor totally consistent. In this regard, the management of uncertainty plays a very important role. [Zimmermann, 2001] Using fuzzy rules, which are means for depositing this information, are found more suitable than using traditional crisp concepts.

The representation of imprecise knowledge demands adequate reasoning methods. On the ground of the fuzzy logic, the fuzzy inference mechanisms were developed. The two main algorithms are known as the *Mamdani – Assilian* and the *Takagi – Sugeno* algorithm. Both of them operate on the fuzzy rules.

If the antecedent is true (in Boolean logic), then the consequent is applied. Nevertheless, if rules are described as fuzzy concepts, the rule antecedent usually is not consistent with the observed fact. The Boolean principles are generalized in the fuzzy systems based on α-level. [Niskanen, 2004] If the antecedent is evaluated as true to definite degree, then the consequent is applied to the same degree. Generally true, that the fuzzy inference process consists of five main steps [Křemenová, 2004]:

- Fuzzification of inputs – the degree, to which the input belongs to the appropriate fuzzy set must be defined.
- Application of fuzzy operator, if the antecedent of the fuzzy rule has more parts.
- Application of implication method, taking the waight of the rules into account. It means in practice reshaping of the resulting fuzzy set.
- Aggregation of outputs of all rules into a single fuzzy set.
- Deffuzificataion – determining single value from the fuzzy set. Several methods were introduced, from which the most common is to assign the centroid of the fuzzy set to the result.

The result of each fuzzy inference is clearly a fuzzy set. This set can be converted to a single real number, if this is needed, by a defuzzification method. The outcome of any deffuzzification of a given fuzzy set should be the best representation, in the context of each application, of the elastic constraint imposed on possible values of the output variable by the fuzzy set.

4.3.1 Mamdani – Assilian inference system

The Mamdani's fuzzy inference method is the most commonly used fuzzy methodology. [Matlab manual] The one of the first control systems, which was built and which used the fuzzy set theory, was just the Mamdani's method. In 1975, it was designed by Ebrahim Mamdani as a trying to control a steam engine and boiler combination, which was synthesizing a set of linguistic control rules in terms of experienced human operators. The whole Mamdani's procedure was based on Lofti Zadeh's paper from the year 1973, concerning fuzzy algorithms for complex systems and decision processes.

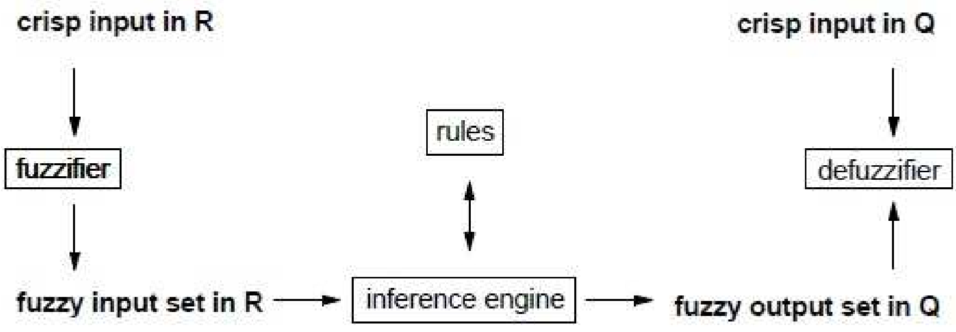


Figure 4.3: Mamdani – Assilian fuzzy control system.

The Mamdani-type inference awaits the output membership functions to be fuzzy sets. There is a fuzzy set, after grouping process, for each output variable that has to be defuzzified. It is possible and many times more effectively, to use a single spike as the output membership function rather than a distributed fuzzy set. At some point, this type of output is known as a singleton output membership function and it can be considered as pre-defuzzified fuzzy set. It improves the efficiency of the fuzzification process, because the computation of more general Mamdani method, which is demanded, is much more simplified. In addition, the Mamdani method finds the centroid of a two-dimensional function.

The Mamdani fuzzy inference system consists of a fuzzifier, fuzzy rule base, an inference engine and a defuzzifier. It is shown in Figure 4.3.

Conventional fuzzy control systems require crisp outputs to result from crisp inputs. The Figure 4.3 shows how a crisp input in set of real numbers (*R*) can be operated on by a fuzzy logic system to yield a crisp output in set of rational numbers (*Q*). This Mumdani system is realised using the following steps.

- **Fuzzification of inputs**

  The fuzzifier maps crisp input numbers into fuzzy sets. The value between 0 and 1 each input is given represents the degree of membership that input has within these output fuzzy sets. Fuzzification can be implemented using look-up tables or using membership functions.

- **Application of fuzzy operators**

  In the case where multiple statements are used in the antecedent of a rule, it is necessary to apply the correct fuzzy operators. This allows the antecedent to be resolved to a single number that represents the strength of that rule.

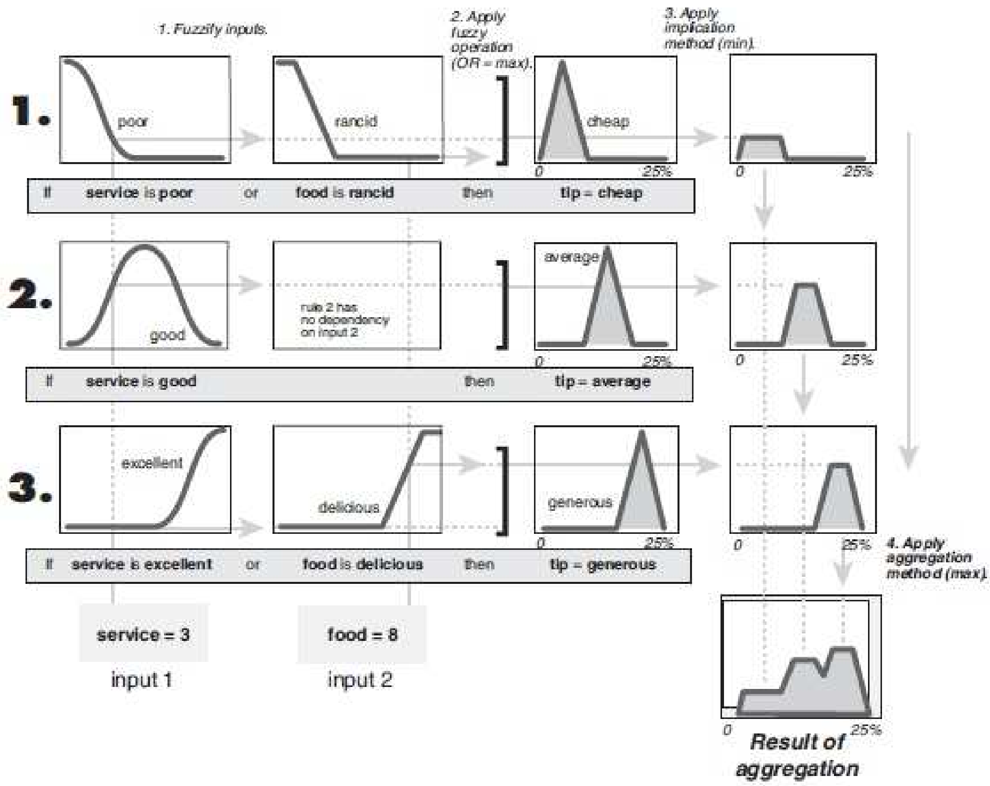


Figure 4.4: Diagram showing aggregation and defuzzification**.** [Matlab manual]

- **Application of implication method**

  This part of the Mamdani system involves defining the consequence as an output fuzzy set. This can only be achieved after each rule has been evaluated with a weight (a number between 0 and 1), which is applied to the number given by the antecedent, and it is allowed contribute its weight in determining the output fuzzy set.

- **Aggregation of all outputs**

  Because decisions are founded on the testing of all of the rules in a fuzzy inference system, the rules must be combined to a certain extend in order to make a decision. The fuzzy outputs of each rule need to be combined in a meaningful way to be of any use. Aggregation is the method used to perform this by combining each output set into a single output fuzzy set. The order of rules in the aggregation operation is unimportant as all rules are considered. The three methods of aggregation available for use include *sum* (sum of each rules output set), *max* (maximum value of each rule output set) and the *probabilistic OR* method (the algebraic sum of each rules output set). An example of aggregation process using the max operator can be seen in Figure 4.4.

- **Defuzzification of aggregated output**

  The aggregated fuzzy set found in the previous step is the input to the defuzzifier. As indicated in the model shown in Figure 4.3 this aggregated fuzzy set in set of rational numbers (Q) is mapped to a crisp output point in a set of rational numbers. This crisp output is a single number that can usefully be applied in controlling the system. A number of methods of defuzzification are possible and these include the mean of maximum, largest of maximum, smallest of maximum and centroid (centre of area) methods. The centroid method is the most widely used and can be seen in Figure 4.4.

4.3.2 Takagi – Sugeno inference system

The fuzzy inference process discussed so far is Mamdani's fuzzy inference method, the most common methodology. This section discusses the Takagi – Sugeno method of fuzzy inference. The Takagi – Sugeno fuzzy model was first introduced in 1985 and in many respects it is similar to the Mamdani method. The first two parts of the fuzzy inference process are exactly identical; these parts are fuzzifying the inputs and applying the fuzzy operator. The main difference between the Mamdani – Assilian and Takagi – Sugeno methods is that the Takagi – Sugeno output membership functions are either linear or constant [Sugeno, 1985]. The output membership functions can be thought of as singleton spikes that submit to a simple aggregation instead of other aggregation methods such as *maximum*, *probabilistic OR method* or *sum*.

A typical rule in Takagi – Sugeno fuzzy model has the form:

If Input 1 = x and Input 2 = y, then Output is $z = ax + by + c$

For the zero-order Sugeno model, the output level z is a constant, because $a = b = 0$.

The output level z_i of each rule is weighted by the firing strength w_i of the rule. For example, for an *AND* rule with Input 1 = x and Input 2 = y, the firing strength is

$$w_i = AndMethod\left(F_1(x), F_2(y)\right)$$

where $F_{1,2}(.)$ are the membership functions of Inputs 1 and 2. The final output of the system is the weighted average of all rule outputs, which is computed as:

$$FinalOutput = \frac{\sum_{i=1}^{N} w_i z_1}{\sum_{i=1}^{N} w_i}$$

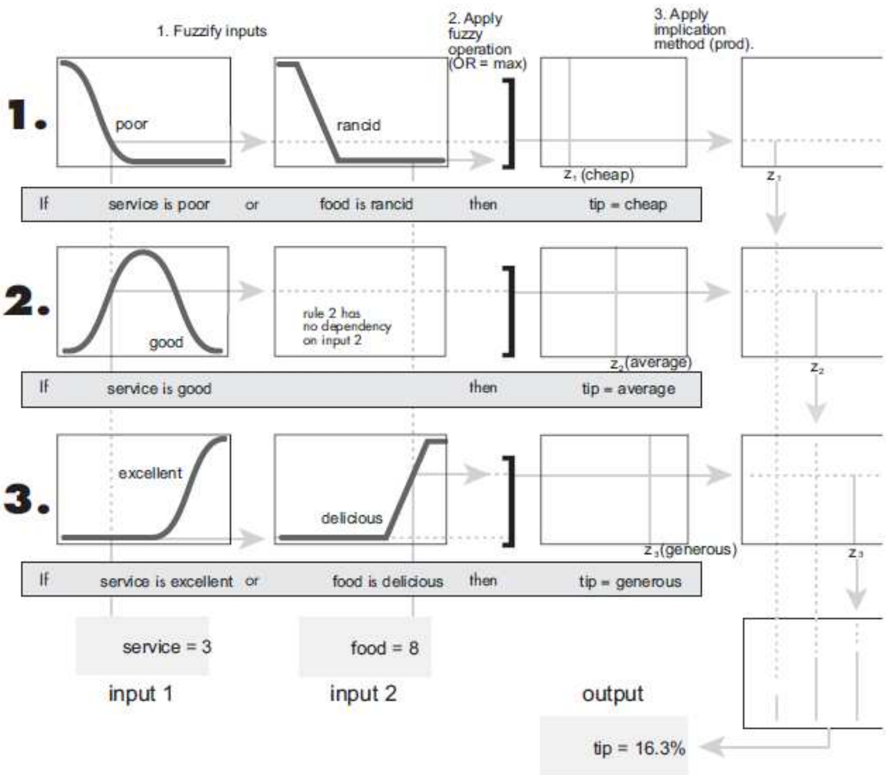


Figure 4.5: Implementation of Takagi – Sugeno model. [Matlab manual]

Figure 4.5 shows the application of basic rules for a Takagi – Sugeno model, all rules have been written using the *"or"* connector, for example:

If Input 1 = x or Input 2 = y, then Output is z

The Takagi – Sugeno fuzzy inference system is computationally efficient and its ability to interpolate multiple linear models makes it particularly suited to modelling non-linear systems.

4.3.3 Comparison of Mamdani – Assilian and Takagi – Sugeno inference system

As has already been noted, in terms of inference process, there are two main types of fuzzy inference systems: *the Mamdani – Assilian type* and *the Takagi – Sugeno type*. In terms of use, the Mamdani fuzzy inference system is more widely used, because it provides reasonable results with a relatively simple structure, and also owing to the intuitive and interpretable nature of the rule base.

Since the consequents of the rules in a Takagi – Sugeno fuzzy inference system (FIS) are not fuzzy, this ability of systems to cooperate effectively and reciprocally. However, since the consequents of the Takagi – Sugeno FIS's rules can have as many parameters per rule as input values, this translates into more degrees of freedom in the design than a Mamdani FIS, so thereby the systems's designer has providing more flexibility in the design of the system. [Jang, 1993] However, it is necessary to be noted that the Mamdani FIS can be used directly for multiple input – single output systems as well as for multiple input – multiple output sytems. Whereas the Takagi – Sugeno FIS can be used only for multiple input – single output systems.

The Mamdani FIS can be seen as a function that represents the system's input field into its output field. This fact ensures that there exists a Takagi – Sugeno FIS that can approximate any given Mamdani FIS with optional level of precision.

The main results from comparison the Mamdani FIS and with the Takagi – Sugeno FIS are:

- The Takagi – Sugeno FIS is more flexible, because it provides more parameters in the output and while the output is a function of the inputs, it expresses a more explicit relation among them.
- In terms of computation, the Takagi – Sugeno FIS is more effective because the process of complex defuzzification of the Mamdani FIS is substituted for a weighted average.
- With regards to the structure of the Takagi – Sugeno FIS rule outputs, it is more suitable for functional analysis than the Mamdani FIS.

From the above mentioned, it seems that the Takagi – Sugeno FIS is always more effective than the Mamdani FIS. But in many branches it is used just the Takagi – Sugeno FIS. Here come out the question: "why they developed with a Takagi – Sugeno FIS?" There exists a main reason:

- It does not make any sense to group different nature outputs with a weighted average, for classification problems, where the rules outputs are independent of each other (that means multiple input – multiple output systems). Nevertheless, in Mamdani FIS it makes sense to select the output with the best assign (min-max inference). Takagi – Sugeno FIS is suitable for multiple input – single output problems, that means systems with the same output linguistic variable. It is matter of fact, that any multiple input – multiple output system can be divided into several multiple input – single output systems.

The following are some considerations about the different Mamdani – Assilian and Takagi – Sugeno methods. [Matlab manual]

Advantages of the Takagi – Sugeno methods:

- It is computationally efficient.

- It works well with linear techniques.
- It works well with optimization and adaptive techniques.
- It has guaranteed continuity of the output surface.
- It is well suited to mathematical analysis.

Advantages of the Mamdani – Assilian method:

- It is intuitive.
- It has widespread acceptance.
- It is well suited to human input.

Mamdani – Assilian method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, the Mamdani – Assilian type fuzzy inference entails a substantial computational burden.

On the other hand, Sugeno method is computationally effective and works well with optimisation and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.

Chapter 5

Geological map

5.1 Geological map and its plotting

In terms of cartography, any geological map belongs to maps, where the topographic background is complemented with any other specifications. For geological maps, the topography is replenished most often with data, which contain extensions of different kinds of rock and geological systems. It is very complicated to define the geological map, but if all details and specifications are not taken into account, *the geological map is miniaturized and generalized image of geological situation, and it is represented on an appropriate topographic background*. [Pouba, 1959] That means, if variant rocks, stratas, faults and other geological phenomena, which can be observed on the land surface, are put in topographic map, the result is geological map.

The geological map shows the distribution of various types of bedrock in an area. It usually consist s of a topographic map (a map giving information about the form of the earth's surface), which is shaded or coloured to show where different rock units occur at or just below the ground surface. In Figure 5.1 it is shown the geological map of an area in the Liberec Region (to the west from Liberec). It tells us for instance that there is abundance of loess loam, deluvial sediments, granite and sandy loam. Lines on maps are drawn to show the boundaries between each of the rock units.

The geological map is a special map made to show geological features. Units of rock or geological strata are illustrated with colour or symbols, which represent, where they are exposed

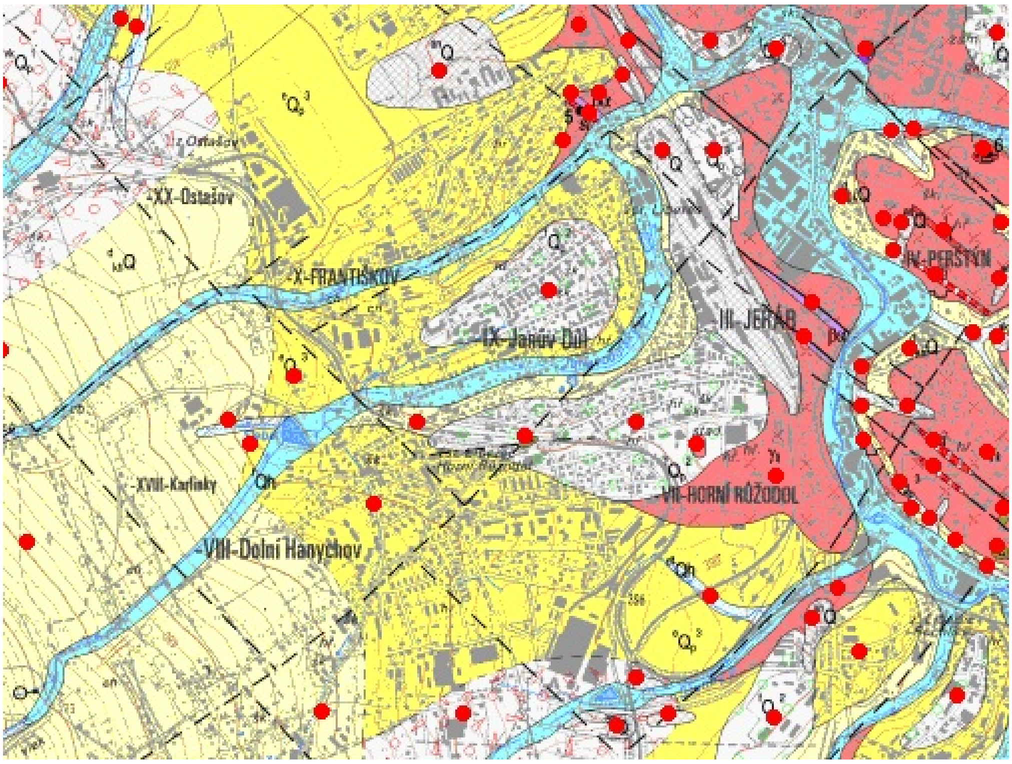


Figure 5.1: A geological map of part of the Liberec Region.

at the surface. In the geological maps are displayed information about the geological bedrock, its structure, composition, data on groundwater, data on mineral deposits and other.

Geological maps provide a quick way to identify and understand the geology and geological characteristics for the requested location. Geological maps serve not only geologists, but they are also used in other sectors. Geological maps are nowadays a part of geographic information systems, so it is possible to combine them with other data and can make and receive outputs from complex analysis.

Geological maps show the locations of the rock and the stratigraphic units in different colours or hatches. In addition, it is possible to find there much more information. According to the scale, the maps are subdivided further more. In geological maps can be found for example which rocks compose the bedrock and how old they are, various failures such as faults or nappes, dip of layers (if the rock has obvious stratification) or the location of important geological localities. Geological maps originate during the geological mapping, which is a very complex process (preparatory work, mapping, the processing of obtained data and the creation of maps). Geological maps can be divided into uncovered and covered maps. Covered geological maps contain Quaternary cover of bedrock in basement, if it is the thickness is more than 2 meters. Uncovered geological maps represent only the bedrock that means without Quaternary cover. The geological maps can be also divided according to the scale, from general to very detailed maps (according to the scale there are posed claims and different criteria while the mapping). Further, the geological maps can be divided according to content (on which phenomenon connected with geology are focused):

- geological maps (they show the geological bedrock and the structure in terms of rock types and their age, they provide information on geological structure of territory),
- engineering geological maps (they evaluate the geological background in terms of demandingness of foundation building engineering),
- hydrogeological maps (they evaluate the geological environment in terms of permeability and groundwater flow, groundwater quality, abundance and type of collector and also in terms of variability of these phenomena),
- maps of soil cover (they divide the area according to the evolution of the soil profile into the soil types),
- maps of deposit (they illustrate the abundance of deposit of mineral resources),
- structurally – geological maps (they show structural systems at the expense of other geological phenomena),
- maps of documentation points (they originate during the mapping and into them are drawn position of documentation points).

This list of special maps is full of only examples of geological maps, not of all existed maps; there are still many other special maps.

Because geological maps show the stratigraphy and relative age of formations, it is possible to derive whether strata, which are illustrated on the map, are older or younger than their neighbouring systems. If this is known, it is possible to gain the insight into the general structure of the geology of the area by looking at the structure of outcrops in relation to the geography of the landscape. To the geologist it can be enable to draw cross sections through areas to show the underground geology along that line of section.

Often these maps seem to be of little value to the engineer and can seem to be quite misleading. Many lithological terms are contained in names of system, such as limestone or sandstone, which were given at the time, when it seemed to the geologist discovering the system that the dominant rock was limestone or sandstone. Since the original discovery the name of system could have been applied to strata of similar age but with different lithology. It must also be understood that the majority of geological maps were made with the scientific aim of explaining the geology rather than for use in engineering practice. Therefore, most of the older maps do not incorporate much detail of superficial geomorphological features, such as landslides.

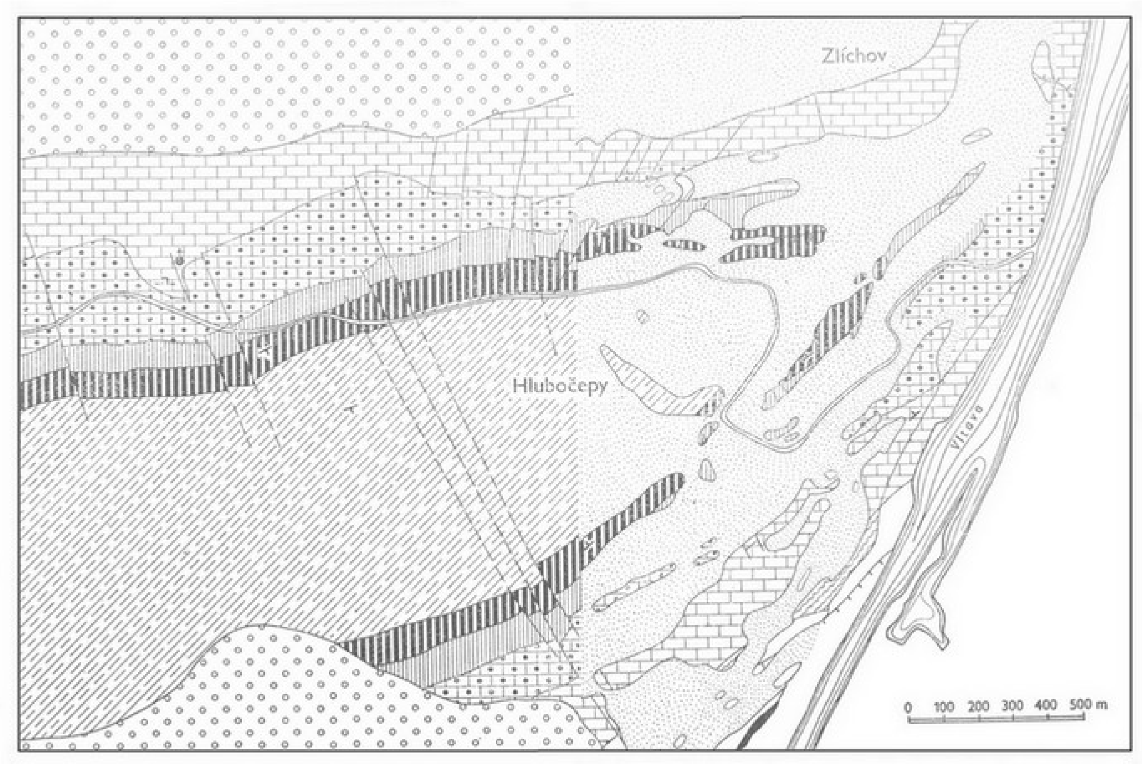


Figure 5.2: Geological map, where left side is uncovered (without surficial systems), the right side is covered (with surficial systems). [Pouba, 1959]

If in the map has expressed the geological situation, which is in the reality, that means in which are marked surficial systems and bedrocks, then this map represents the Quaternary cover. Also, it is called the covered map. If it is not displayed, then the map is uncovered, and thus this map represents the geological situation in certain depth and there is not displayed the Quaternary surficial system, see Figure 5.2. In addition to geological maps of the Earth's surface there are produced the subsurface maps (for example in mining districts), which show the geological situation at a certain depth below the surface. If the map shows the geology Earth's surface in a period of geologic history, it is called the paleogeological map. The geological formations are indicated in the Figure 5.3, taken from [Pouba, 1959].

The geology is represented in geological maps by colour, lines and special symbols, which are unique to geological maps. Understanding these features will allow understanding much of geology shown in almost any standard geological map.

The most striking features of geological maps are its colours. Each colour represents a different geological unit. A geological unit is a volume of a certain kind of rock of a given age range. The colour representation of rocks in geological maps is governed by steady customs. Hold generally, that certain colours are used for specific types of rock or ages of rock. Analogous to water course in maps pictured with blue colour, the granite rocks are marked with red or basic

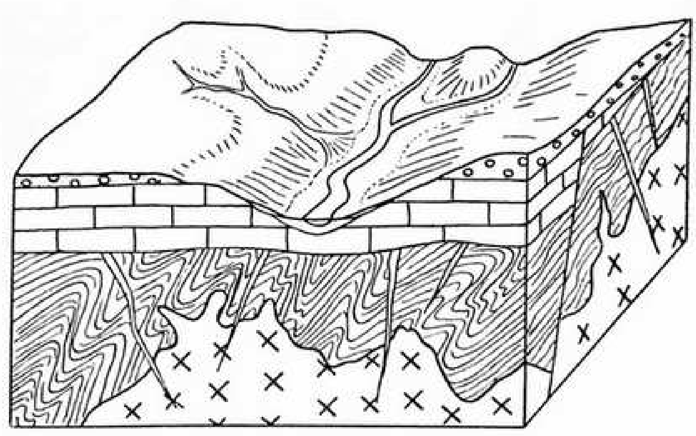


Figure 5.3: Image of geological formations. [Pouba, 1959]

eraptive rocks are displayed mostly with dark green etc. Please note that this applies only generally, but it is recommended to look at the legend, because for example red colour can mean something else in various maps.

Geological units are named and defined by the geologists who made the geological map, on the basis of their observations of the kinds of rocks and their investigations of the age of the rocks.

In addition to colour, each geological unit is assigned a set of letters to symbolize it on the map. Usually the symbol is the combination of an initial letter with one or mare small letters. The capital letter represents the age of the geological unit. The capital letter represents the age of geological unit. The geologists divide the history into Eras, Period or Epochs, mostly on the basis of fossils found in rocks. The most often used division of time is to the Period. Occasionally the age of a rock unit will span more than one period, if the period of many years required to create a body of rock happens to fall on both sides of a time boundary. The few geologic units formed an unknown amount of time ago have letter symbols with no capital letters.

The place, where two different geological units are found next to each other, is called a contact and this place is represented by a different kind of lines on the geological map. The two main types of contacts shown on most geological maps are depositional contacts and faults.

All geological units are formed over, under or beside other geological units. For example lava from a volcano flows over the landscape, and when the lava hardens into rock, the place, where the lava-rock rests on the rocks underneath, is a depositional contact. Where the original depositional contact between geological units is preserved, it is shown on the geological map as a thin line.

On the geological map the faults are expressed as more or less straight lines, which can be simple or can variably branch forth. According to these lines the terrain structure used to be faulted, and that is expressed by their abrupt interruption. [Pouba, 1959] The tectonic geology is concerned with classification of these movements, like different kinds of dip-slip faults, centrifugal faults, dislocations, etc. On the map these faults are drawn in as a continuous line if the faults are known for a certainty and verified, or as dash line, if the course is only assumed.

Another kind of line shown on most geological maps is a fold axis. In addition to being moved by faults, geological units can also be bent and warped by the same forces into rounded wavelike shapes called folds. The line that follows the crest or trough of the fold is called the fold axis. This is marked on a geological map with a line a little thicker than a depositional contact, but thinner than a fault.

All thicknesses of lines are also modified by being solid, dashed or dotted. Often contacts are obscured by soil, vegetation or human construction. Those places where the line is precisely located it is shown as solid, but where it is uncertain it is dashed. The shorter the dash means the more uncertain the location. A dotted line is the most uncertain of all, because it is covered by a geologic unit, so no amount of searching at the surface could ever locate it. The lines on the map may also be modified by other symbols on the line, which give more information about the line. For example faults with triangles on them show that the side with the triangles has been thrust up and over the side without the triangles, that kind of fault is called a reverse fault or a thrust fault. All the different symbols on the lines are explained in the map legend.

Many kinds of rock form in broad, flat layers, called beds. In some areas thick stacks of rock beds that have built up over millions of years remain in their original flat orientation, where they can be viewed as multicoloured horizontal layers of rock.

Tilted beds are shown on a geological map with a strike and dip symbol. The symbol consists of three parts: a long line, a short line and a number. The long line is called the strike line and shows the direction in the bed that is still horizontal. Any tilted surface has a direction that is horizontal.

The strike line shows that horizontal direction in the beds. The short line is called the dip line and shows which way the bed is tilted. The number is called the dip and shows how much the bed is tilted, in degrees from flat. The higher the number, the steeper the tilting of the bed, all the way up to 90 degrees if the bed is tilted all the way onto its side. Strike and dip symbols can be modified to give more information about the tilted beds just like lines can be. All modifications are explained in the legend as well.

5.2 Geological mapping

The development of geological mapping process in area of the Czech Republic is bound up, especially in the beginning, with development of geological research in neighbouring states, mainly in Germany, Austria and Hungary. In the late eighteenth century, also in our country occurred the development of inorganic sciences, especially *geognosie*, how geology was called at that time the geology, owing to A. G. Werner and his schoolchildren. A. G. Werner acted in Freigeberg in Saxony and in 1786 issued in Prague the work "*Classification und Beschrebung der Gebirgsarten*", where are distinguished four geological units: crystalline schists, in principle Palaeozoic, late Palaeozoic until Tertiary, late alluvia. This dividing of layers was used in most of the geological work, including the geological maps.

The Werner's predication, even if it was not true in some opinions, has got abroad round the world. His distinction of earth layers, that could create geological maps from petrographical maps, was not used on the latest geological maps.

In 1849 the floatation of Viennese geological institution had big significance for research of our country. At the head of the institution was capable W. v. Haidinger, whom succeeded to map whole Austria – Hungary as a whole in about 14 years at the scale 1 : 144 000. Bohemia was mapped in the years 1853 – 1862.

From that time the geological mapping at the area of not only our country passed through progression, which leaded to better and more accurately information about geological bedrock.

Nowadays the Czech Geological Survey is concerned with basic geological maps at the scale 1:25 000, and the plan is to map the whole Czech Republic, but it will take a long time. After completion coverage of the Czech Republic with maps at the scale 1 : 50 000 was renewed from the year 1996 the geological mapping at the scale 1 : 25 000.

The basic geological mapping process is the primary cognitive method of the geological structure and the development of the Czech Republic area. During the basic geological mapping process it is a benefit derived from field work, archive data, laboratorial data and findings from other geological disciplines.

The basic geological map is asserted especially [Hanžl, 2009]

- for geological research and survey, protection and searching minerals, the origins of underground water and usage of geothermal energy;

- for research and evaluation of the influence the geofactor to the environment and strategic planning the land utilization and natural resources, for solving the ecological problems and protection the unanimated nature;
- for the assessment of geological conditions for notable buildings and for assessing the influence of building-up on the rock environment;
- for the solution of theoretical and applied as well geological problems;
- for drawing general and thematic geological maps and for regionally geological synthesis;
- for drawing derivative and purpose maps and as the basic for pursuance of special and detailed geological researches;
- for popularization and for development of geology and other Earth's science.

The basic geological map as summarily piece, which describes significant characteristics of rock and living environment, is applied for the general lay and skilled public.

Geological mapping processes and documentation in the landscape is recorded into the basic maps, which are larger scale than the final geological map, e.g. for the final geological map with scale 1:25 000 are used maps with scale 1:10 000, but in some special cases into the more detailed topographic basis.

The objects drawn in geological map must be backed up by geological documentation. In the basis geological map are represented geological entities with the surface of at least 5 mm^2 or which are wider than 1 mm in the map itself. If the entity is smaller than the previous size, but it is necessary to draw it because of its geological importance, it is extended into the given proportion.

The main source data used for mapping are a basic topographic map, an aerial photograph, and a soil map. The character of soil is derived among other things from the character of sub-surface as well. But the character of soil also depends on climate, ground water level, anthropogenic activity, vegetation and others, thus it is not possible to map geology on the basis of soil. In many aspects the soil character can help. For example in some areas the sandy soils are evolved on the fluvial sand and gravel, strongly humos soil (clay) are evolved on the impermeable sub-surface (e.g. on chalk clay and marlite).

Soil is not depicted in the geological maps, but only geology, whether with or without Quaternary. The basic topographic map is one of the most important grounds for geological mapping. Further, the information on the basic map reflects the terrain shape, important for predicting the soil and geological trend and it is often used for positioning.

Another important piece of information to define the geology is the historical geological mapping of the area. Before mapping in terrain it is appropriate to become acquainted with the given territory. The help for familiarization with the territory forms previous older maps. Nowadays the whole country is covered by geological maps at a scale of 1:50 000. Other older maps can be special maps in a larger scale, for example in urban conglomerations the special maps could be engineering geological maps at the scale of 1:10 000 and some are 1:5 000 as well. Another information source is data from documentation of older geological work, such as geological reports, holes, groundwater reports, calculations of mineral reserves, etc.).

Further more the source for more information forms Earth observation techniques. From the aerial photographs, one can observe the morphology of the given area. The morphology is derived from geological composition, such as tectonics, resistance of rocks, Quaternary sediments. The aerial photographs are important particularly for mapping the Quaternary, e.g. on the photographs are clearly recognizable the alluvial cones, contemporary alluvial streams, which are filled with Holocene fluvial sediments. On the basis of the morphology it could discover, where the earth slides might be situated.

According to the morphology, to the drainage pattern and drainage of valley also it is possible to detect the main directions of tectonic structure, like faults. Quite noticeably and suspicious is the long-unchanging direction of the valley of the stream or river, or when at the district are evolved parallel valleys, which have the concurrent direction. Then it can be assumed, that these valleys are tectonically predisposed. Do not forget, that the Earth observation technique is a helpful tool, with which can be found some characteristics of the terrain, but everything is necessary to check in the reality in the terrain. [Beazley, 1981]

The geological mapping in terrain requires for geologist to have at least a rough image about the structure of the area. Except for the earlier mentioned necessaries, the geologist needs to see his domain. The geologist must think about orientation hikes and align them in the map. The purpose of these hikes is to obtain as best and complex view of geology of the terrain as possible, so it is necessary to dislocate equally to go through different terrain arrays and to realize such a maximum of amount of rock exposures. The orientation tour should pass through uncovered terrain, if possible, where the larger segments of rock can be observed without being covered by later and surficial formations. The tracks should lead across directions of main layers or across axis of main folds in terrain.

Orientation hikes should pass through different terrain systems (combs of mountain, slopes and valleys) and afford easy orientation. Large amount of exposures provides to detect main directions and slopes of layers, to define main types of rocks and to appoint criteria for their distinguishing. For more detailed touring geologists proceed in order that each layer boundary, which was proved during orientation tours, to contact gradually into fluent line. During

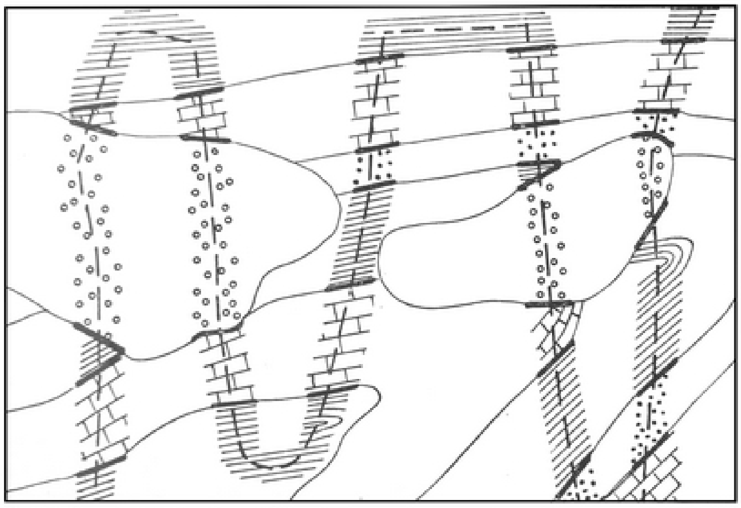


Figure 5.4: Orientation hikes through directions of layers. [Pouba, 1959]

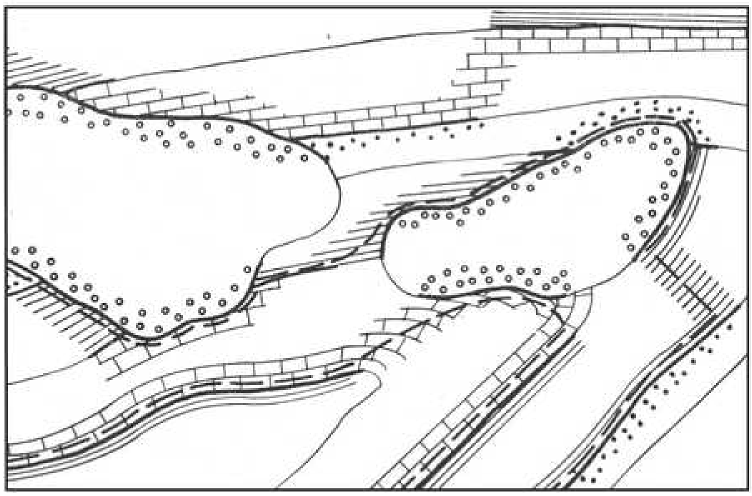


Figure 5.5: Orientation hikes along the geological boundaries. [Pouba, 1959]

mapping the boundaries between layers in terrain can pass by dual ways: cross tour (Figure 5.4), or longitudinal tour (Figure 5.5).

During touring the geologist searches outcrops of rock. The outcrop represents rock, fold, cut. On the outcrops are recorded description of lithological character of rock, its structure (measured direction and layer dip for sediments, or metamorphic foliation, lineation for metamorphic rock, direction and character of joints or of fault plane. Some patterns of rock, if necessary and very rarely, are tested in laboratories.

In some areas, for example agricultural areas or badly permeable terrain, the classification of determining boundaries or definition of the rock type may be difficult, so this wholly depends on knowledge and ability of the mapper. The mapper is forced to simplify shapes of polygons and omit some details, because he generalizes from reality to map with a scale smaller than 1:1. Much information obtained in the field is left out.

5.3 Uncertainty in geological mapping

The mapping process of geology is entirely manual, based on expert interpretation and a great amount of estimation is necessary. The geological maps carry uncertain information, which was propagating during every step of the map production and the task is to describe it.

For proper understanding of the content of the map the knowledge about the uncertainty is essential. If an agreement on how to express the map quality can be reached, then the information should be documented and released to users. [Longley et al., 1999] However, current geological maps do not demonstrate any information like this. The only sources for the quality assessment are geologists experienced in geological mapping.

Documentation of the map quality is troubling, even if some techniques can be used. Generally, inaccurate features are depicted on a map by using symbols, which are explained in the attached legend. A geological map is also succeeded by reports and explanation notes, which represent extensive textual description serving to specify and to expand information, and to clarify each item in the legend. The explanation notes recapitulate total important geological information, which were obtained during the geological mapping of the map sheet. [Hanžl, 2009]

A control over the imprecision in geology enables the fuzzy set theory. Additionally, such representation of geological phenomenon depicts the soft transition between geological classes in a more realistic way. The knowledge geologist possess about geological maps and the uncertainty, that is contained in them, has to be documented. The knowledge acquisition is pretty difficult, because of the complexity of the problem and because of the large amount of the affecting factors.

For providing the comprehensive information to the users it is necessary to use systematic mapping technique and documentation. For map relationship, the mapping process is provided not only in the area of the map sheet, but it interferes in the neighbouring “map sheets”, even if the neighbouring map sheet is already mapped and published. Here applies the following: the previous published map (in the same addition) does not have to be true.

The main problem in depicting geology is the classification. Geological basement is in many cases not visible and in other cases fragments are detected in the soil at the surface. The geological mapping is based on discovering the boundaries between different geological units; concept of classification affects also the positional accuracy.

The classification brings along also the ambiguity. The water areas, although they are not geological units, are also presented in the geological maps. Also generalization is the problem of all maps. Many details have to be omitted during mapping and in addition much information is left out during the map production. Despite of that, such details can be of great importance for the users. Study of the information, which does not occur in the final map, could help with modelling the uncertainty.

The quality of the final map depends greatly on the expert knowledge and experiences of geologists. The geological mapping expertise is based on interpretation of different data sources. The quality of these sources reflects in the quality of the geological map.

Chapter 6

Fuzzy model of geological map

6.1 Problematic of geological mapping

Sometimes in the real world, the boundary between phenomena and features is not absolute; therefore there is some uncertainty and vagueness in drawing and depicting features.

The geologists regard the fuzzy approach, which enables handling of vague information, as the most realistic description of geology. The result will be a new map layer, which will contain fuzzy membership values to particular geological polygons from the original map. The indicators of the uncertainty in the map are fuzzy membership values. The improvement of the geological maps with using the qualification the uncertainty provides the reliability evaluation of the analysis and conduces to better utilisation of the maps.

The assessment of fuzzy membership values is deciding to correct the fuzzy model. In this work the fuzzy memberships are based on the expert knowledge. On the score of the complexity of the problem, it is developed an expert system for simulating decisions of geologists.

Geological mapping consist in location of geological polygon boundaries. In geology there are two levels of problems with assigning the boundaries of geological rocks.

- The transition zone between two or more types of rocks can cause difficulties because of the occurrence of only dendrites of rock or small outcrops, or in the middle

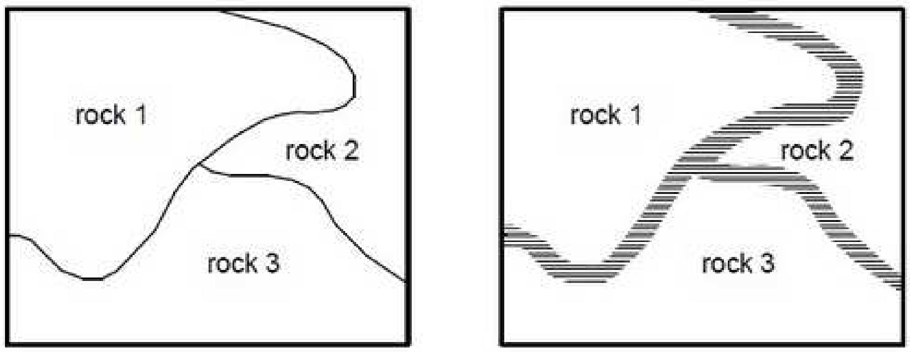


Figure 6.1: (a)Boolean and (b) fuzzy bourderlines of geological polygons.

of geological polygons exist polygons too small and it is not so important to be drawn on the map or the area might not be observed in the field at all;

- The mapping location of tranzition zone and notion it into the map is not accurate, despite of well ability of orientation and usage of modern technology. Eventhough the expert knowledge about definition found phenomena is the best as it can be, the drawings on the map cause great error.

The documentation during mapping geology puts forward notes about location of documentation point, and information about itself. The information predicts the geological structure at any given location. The notes contain descriptions of rock, but the description of all rocks did not occur at the documentation point, thus not even content rocks at the location. Therefore this work deals with the second case, when it takes into consideration only the uncertainty of location.

For one map sheet geologists do the boreholes only in number of first tens, and these boreholes are drilled without conserving the drill core. The boreholes contain cuttings of boring, so it is up to geologist to define the rock under soil layer. These boreholes hold mixtures of types of soil and rock, the borehole is mostly about 1 m deep, then normally gets under the top soil, to the decomposed rock, in such way that it is possible to identify, where the bedrock is formed. The information from the boreholes is based on geologist’s knowledge and experiences.

On the basis of tours and documentation the geologists draw the geology into the map, how they think the boundaries of each polygons of rock type is proceeding through the terrain. There are some places, respectively boundaries of rock types, which are totaly subjective and which are uncoverable with any special documentation or boring and which are not visible at the surface. These boundaries are noticed on the map as suppositios.

Much more complicated than the previously mentioned boundaries of polygons of rock types is the lithological – facies transition. The lithological – facies transition is totally vague, because

Table 6.1: System of sedimentary rocks.

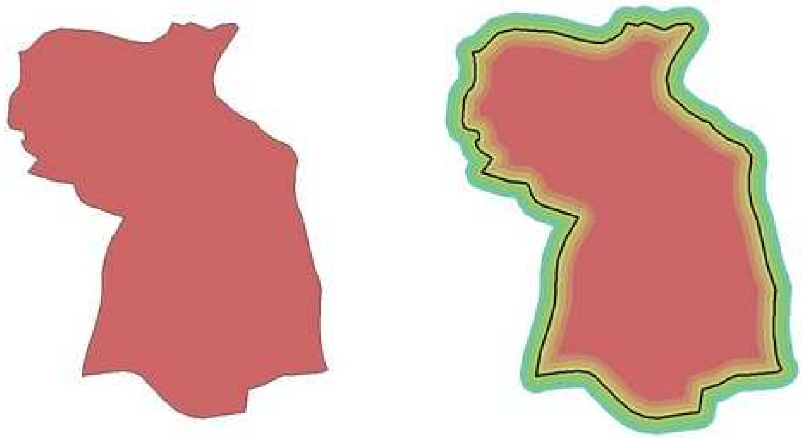
Grain size	Noncoherent rock	Consolidated rock
more than 2 mm	glyder, gravel, boulder	breccia, conglomerate
2,0 – 0,05 mm	sand	sandstone, quartzite
0,05 – 0,01 mm	silt	siltstone
less than 0,01 mm	clay, marl	claystone, marlite

Table 6.2: Membership of each point according to distance from polygon border.

Set of points, which placed in distance (D) from polygon border D = geometric accuracy of map	Membership degree
D > - 3d	7/7 = 1
- 3d < D < - 2d	6/7
- 2d < D < - d	5/7
- d < D < 0	4/7
0 < D < + d	3/7
+ d < D < + 2d	2/7
+ 2d < D < + 3d	1/7
D > + 3d	0/7 = 0

it depends on the grain size. (Table 6.1) This table is only an example of division of the rocks according to the grain size, but each literature determines various division. Therefore the distinction of rock type depends also on the geologist’s thinking and his thoughtfulness. The boundary of polygon of rock type can be located at the place, where it is drawn as well as somewhere else. Therefore the geological boundaries of rock type are consistent to the geologist. Each geologist could find out the boundaries elsewhere and with other shapes.

Membership or non-membership of a point in each polygon is known and which can be 0 or 1. In most of real state, the border between polygons is not precise (Figure 6.1); hence determination of the exact border in order to expression the membership or non-membership in a polygon is impractical and impossible. [Sunila et al., 2004]



(a) (b)

Figure 6.2: Showing and storing of spatial data categorically in vector model (a), using fuzzy logic (b).

Polygonal features are made from lines and linear feature are made from points. For showing and storage of linear and pointed features, the amount of membership or non-membership of every point of map according to the nearest distance from linear and pointed features is also uncertain. Usually at these cases, the amount of membership or non-membership of each point according to the distance from linear and pointed feature is recognized by a membership function. For instance, the possibility of mineralization according to its distance from linear features (Fault) and point features (Mineralization indicator) are mentioned.

In conventional map drawing software and GIS software, the border of polygon features are shown and stored categorically (Figure 6.2(a)).

In the real world, despite the uncertainty in the polygon border, it is possible to model existence vagueness in storing and showing polygon border, by using membership degree. The membership degree can be defined by its distance from the polygon border line. All of the points, which are in the same distance from polygon border, have the same membership degree.

Geometric accuracy of a map is calculated by its scale. For example in 1: 25 000 scale, maps accuracy equals is equal to 7 metres. Thus it is possible to calculate membership degree of each point of map domain (related to distance from polygon line) according to map accuracy. For instance, if the existence vagueness in storing and showing of polygon border presume three times further than map geometric accuracy (21 m), three internal buffers and three external buffers from polygon A border in vector model (Figure 6.2(b), the black line) is drawn. If the map accuracy assumed 7 metres then obtains Table 6.2. The fuzzy membership function in Figure 6.2(b) is presented with Figure 6.3.

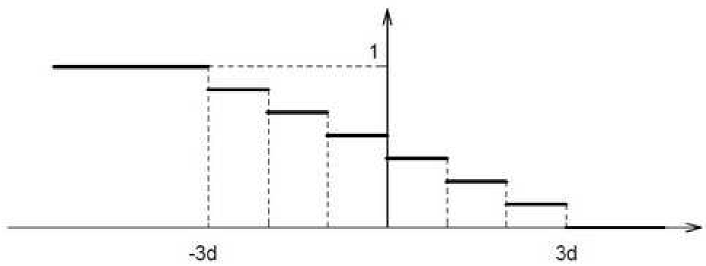


Figure 6.3: Fuzzy membership function of polygon B.

6.2 Problem of colours on maps

There are many possibilities, which colour combination and which hue, saturation or lightness used for map. But not every combination is very useful, readable and clear.

If we look at the colour wheel (Figure 6.4), where colours are arranged according to their chromatic relationship, one can clearly see the complementary colours (Figure 6.5). These complementary colours are located on the opposite side of colour wheel. Complementary colours bring out the best in each other, but when fully saturated complements are brought together, some effects are noticeable. This may be a problem. That is the reason, why it is not suitable to use complementary colours for two different phenomena on a map, for example one for background map and other for symbols. This can bring undesirable effect and illusion .

Every visual presentation involves relationships between figure and background. This relationship will evidence a level of contrast (Figure 6.6). The more of that object, the more visible it becomes. When we create a visual, that are intended to be read, enough contrast between background and text or symbols is important. If there is not enough contrast between figure and ground, a viewer will be confused and this effect may also cause eye fatigue.

When choosing complementary colours, fully saturated colours will offer the highest level of contrast. Choosing from tints or shades within the hue reduces the overall contrast of the composition.

Hypsometric range is a special instance for sequence colour scale. It is often used in physical-geographical maps for the expression of elevation above sea-level. Despite its heavy use it has never been standardized, but there are a large number of modifications. For some users the hypsometric range can be misleading. For example white colour can represent the highest parts of mountains as well as for another user white colour evokes snow, but snowy fields are not located only in the high elevations, for example polar regions are often placed in lowlands. Similarly

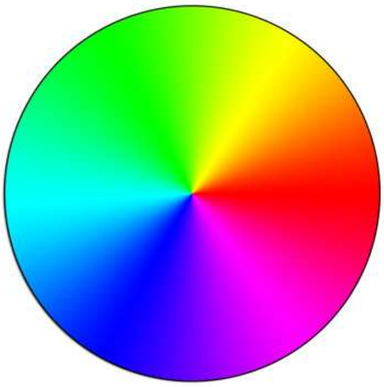


Figure 6.4: Colour wheel.

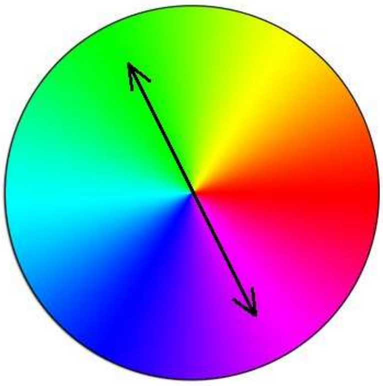


Figure 6.5: Complementary colours.

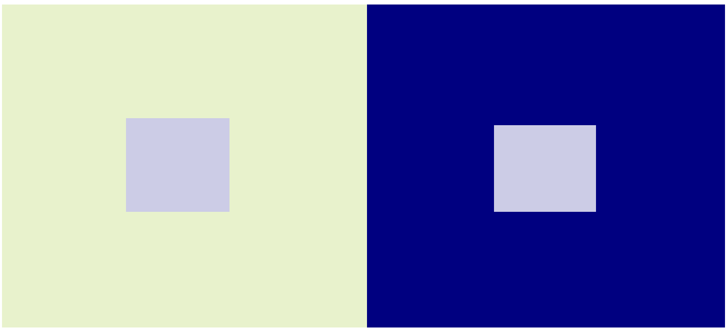


Figure 6.6: Illusion of different colours for the small square inside, even there is the same.

a green colour, which is used for showing lowlands, by a majority of readers it evokes imagination of forest and pasture, which are located in other vertical zones.

The main problem with choice of colour is Xerox effect. When converting a map into monochromatic (black-and-white) form two different colour tone can create on all fours shade of gray colour, by which means the map becomes almost unreadable. These two different colour tones are very often red and blue colours.

6.3 Visualization of fuzziness on maps

There are some possibilities of how to visualize the fuzziness on maps. Some of them were already mentioned earlier. It is very necessary to first think about the map and its visualisation, respectively its colours and how to visualize the fuzziness. Do we have real data of phenomena or do we have expectations with the linguistic variables, which can make rules?

For this work data from geological mapping in a scale 1 : 25 000 was used. But these data were expressed as vector data with special attributes of each vector without documentation of each point, only result of documentation. The area of research was consulted with geologists and other experts, who mapped and worked on this map sheet.

On the boundaries between rock types are depicted by lines which represent a kind of boundary in the geological map. In researched areas occurs 3 types of boundaries between rock types: proved, suppositious and lithological – facies transition. Each of them has different characteristics and different degrees of uncertainty.

The task is to find out the method and technique for projection of the fuzzy logic on the geological map so that the graphic form is intelligible to the user and easily readable.

This problematic is more or less processed and it is well known by the general public, and fully used. It is the coloured hypsometry. The coloured hypsometry is used on the map with small and medium scale, where the contour lines cannot be displayed, but it goes out of contour method, to represent the altimetry with the help of colours. In coloured hypsometry the flats between contours are coloured. The graduation of colours is chosen to evoke the spatial imagination.

The hypsometry represents only one phenomenon, so it can be called "one dimensional". The phenomena express dualistic principle, it shows elevation above sea-level, so from the deep point to the high point. The higher the darker. And for waters: the deeper the darker.

The principle of hypsometry is used in other areas of study as well. For example in the branch of energy modelling of buildings as a thermovision or another example is for weather forecast, whether the forecast of precipitation or temperature, or other anomalies. But always the "hypsometry" is used for one magnitude, which can be expressed as quality and occur between two extremes: *lot of – little*, *high – low*, *hot – cold...*

For geological maps it can be the way, how to "mix" universally the unlimited amount of colour scales.

The principle of coloured hypsometry encounters problems, how to simulate the continuity of reality.There are two main approaches.

- **Hypsometry**, with using the grades of close shades, which are graduated with small distances. This approach is technically practicable. But it quite contradicts the principle of fuzzy logic. In addition this is inflicting the difficulties on the boundaries of each shade, even if steps between intervals appear as coherent. In Figure 6.2(b) some parts seem to be fluent, other disfluent, according to the choice of shades.
- **Continuous colour transition**, so-called *valér* (Figure 6.7). This approach approximates to the fuzzy sets, but it has a rather complicated appearance, which is totally dependent on the colour choice.

The mixture of colours can cause delusions, and not only because of delusions, it is linked with a measure of nonseriousness. The possible starting point, how to represent fuzzy sets and not cause confusion, might be the utilization of black-and-white scale, or scale or one colour with black or one colour with white. The disadvantage of this way is that the map could represent only one scale. It would have to divide into layers, and each layer would represent only one phenomenon, one element.

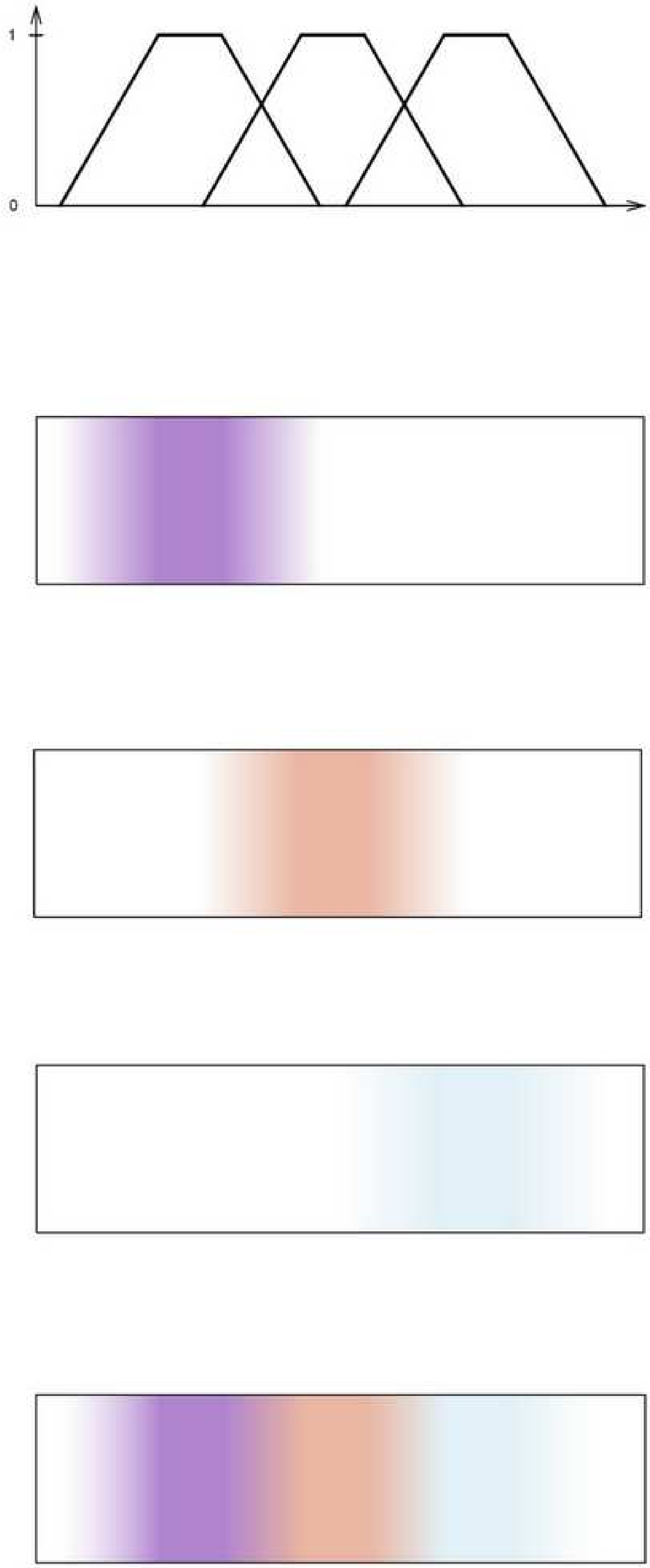


Figure 6.7: Continous colour transition according to the membership function.

Chapter 7

Conclusion

7.1 Conclusion

The main aim of this thesis was to introduce the problems of fuzzy set theory. Today, computers have a good capacity for decision making. Computers use binary logic and hence can only allow for values 1 for true and 0 for false. Statements like “this car is not fast enough” or “this person is quite smart” are rather vague statements which cannot be interpreted by the classical logic.

Fuzzy logic provides an extension from the classical logic for handling the vagueness. Fuzzy logic starts with human language rules. The fuzzy systems convert these rules to their mathematical equivalents. This simplifies the job of the system designer and the computer, and results are in much more accurate representations of the way systems behave in the real world.

Additional advantages of fuzzy logic include its simplicity and its flexibility. Fuzzy logic can handle problems with imprecise and incomplete data, and it can model nonlinear functions of arbitrary complexity. A fuzzy system can be created to match any set of input-output combination. The rule inference system of the fuzzy model consists of a number of conditional “if – then” rules. For the designer who understands the system, these rules are easy to write, and as many rules as necessary can be supplied to describe the system adequately.

Although the geology of every area is different, all geologic maps have several features in common: coloured areas and letter symbols to represent the kind of rock unit at the surface in any given area, lines to show the type and location of contacts and faults, and strike and dip symbols to show, which way layers are tilted. The geology of an area has a profound effect on many things, from the likelihood of landslides, to the availability of groundwater in wells, from the amount of shaking suffered in an earthquake, to the presence of desirable minerals, from the way the landscape is shaped to the kinds of plants that grow best there. Understanding the earth underneath is the first step in understanding the world around us.

Presenting and showing of the real world and its projection on a map has difficulties and problems because of the existence of uncertainty between the spatial objects border, and also the inaccuracy of membership or non-membership for each point of a factorial map according to the distance from spatial objects. Fuzzy logic could be useful and workable in order to prepare a map which shows the existence of vagueness between phenomena and features, which is closer to the real world.

Within the framework of this thesis was the tendency to apply the fuzzy logic to the geological map, but the main problem was the unrecorded data from the mapping process. The method of continuous colour transition is used in Appendix A, but how it is seen, it is not very useful for representation uncertainty in map. Some colours become extinguished or they are not distinguishable. It would be more suitable to use coloured hypsometry or find any other method for diagrammatizing the ruzzy data.

7.2 Future work

Although satisfactory introduction into the problems of fuzzy logic was described, the fuzzy set theory is a large piece of science, and this work is only an outline of this problems. There is still room for improvement in the methods of application into the geography and other natural sciences.

It is obvious that the fuzzy approach to representation of vagueness in geological rocks is more than appropriate. The potentialities of the fuzzy model can be compared to the geostatistical analysis of rocks. The positives of the fuzzy model are its competency with human reasoning and universal applicability.

There is a variety of other geographical features, vague in their nature, which could be effectively represented with the fuzzy models. These are for example water bodies, elevation data, etc. Therefore the fuzzy modelling of other suitable geographical information could be the topic of future research. This thesis is only the introduction, the real and practical application will proceed under the doctoral degree programme.

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Enclosure list

Appendix A

(a) Map slice of geological map 1: 25 000 without rock boundaries.

(b) Map slice of fuzzy geological map 1: 25 000 with using the continuous colour transition.

(c) Map slice of fuzzy geological map 1: 25 000 with rock boundaries from geological map.

Appendix B

CD

/data/

Appendix A

Thesis